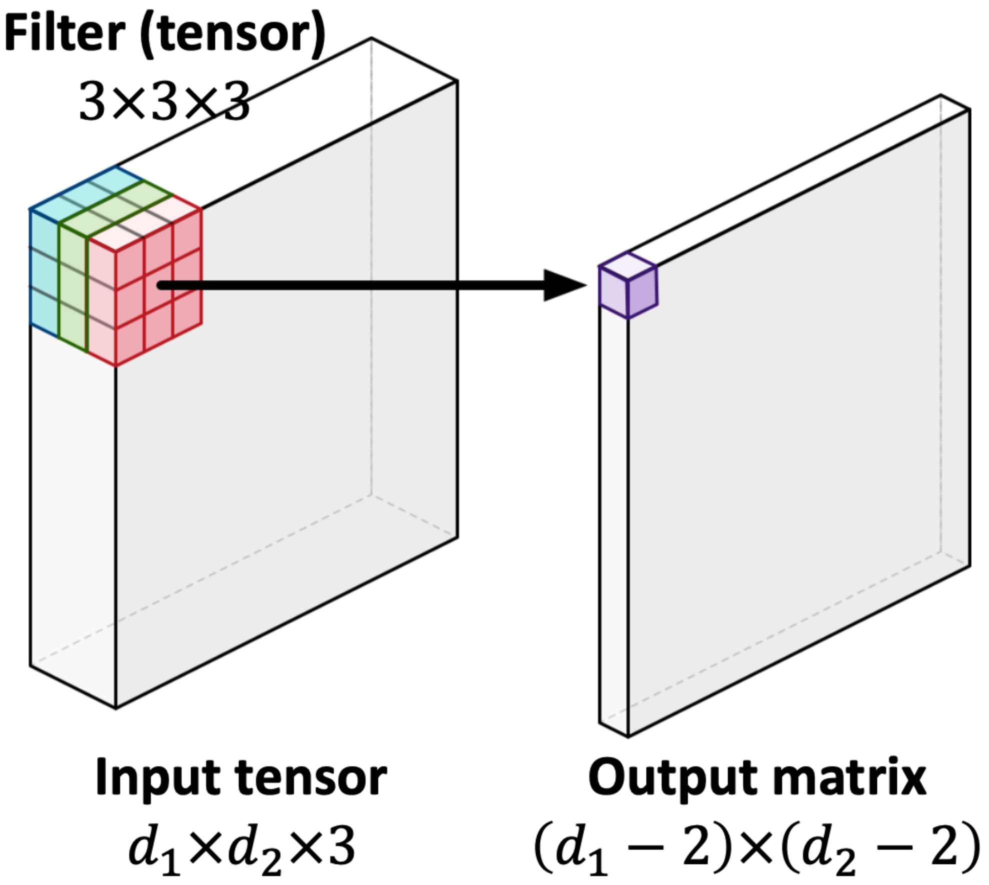
Graph Data and Graph Representation

Neural Networks Design And Application

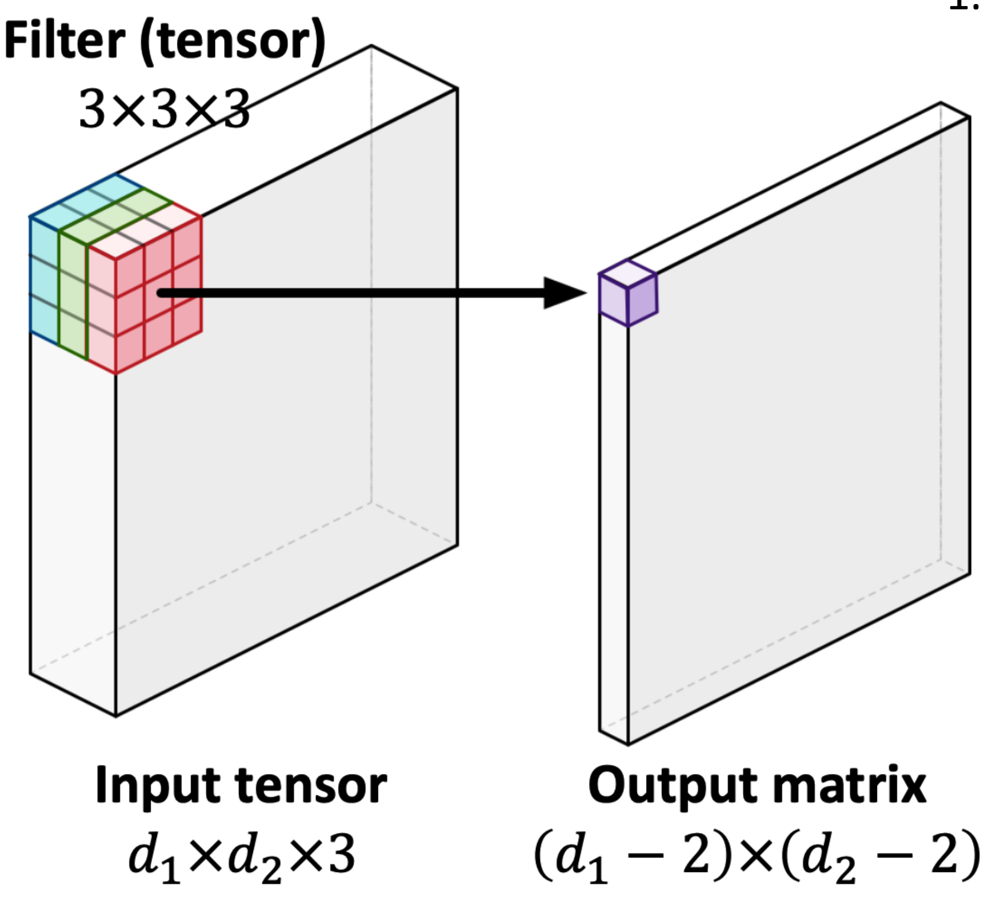
Correlation between data



Correlation between data

Q: is there correlation between:

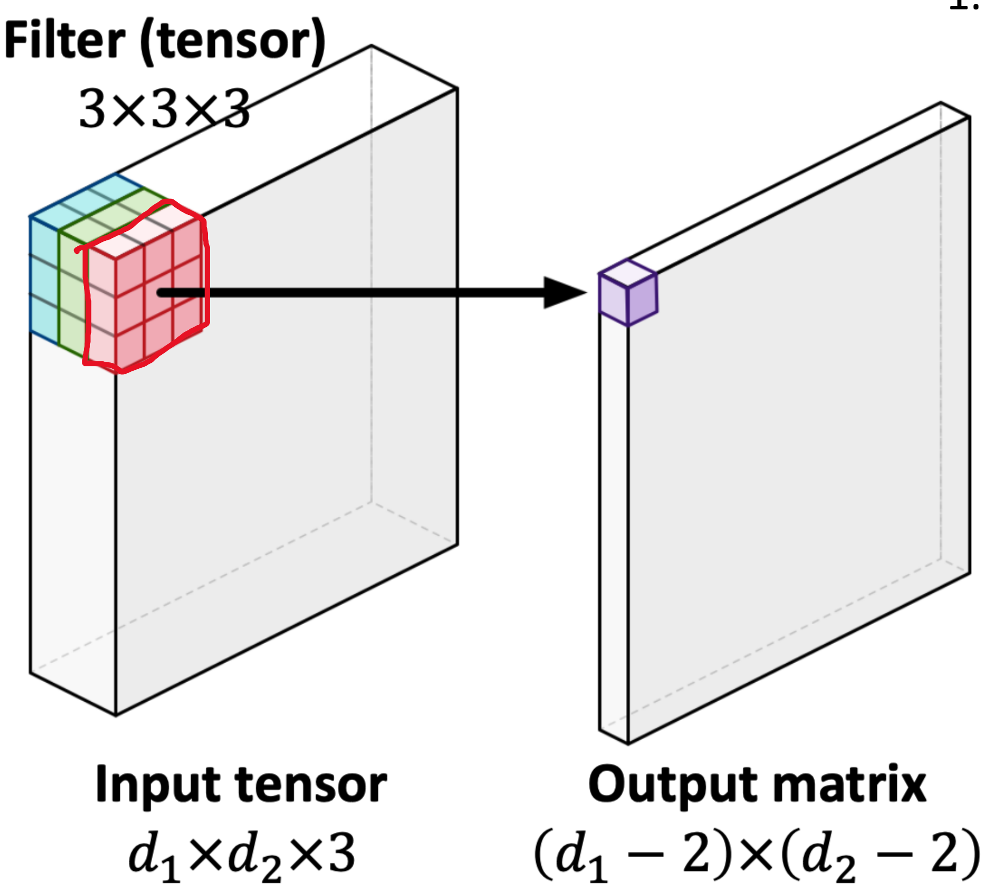
1. locations?



Correlation between data

Q: is there correlation between:

1. locations?

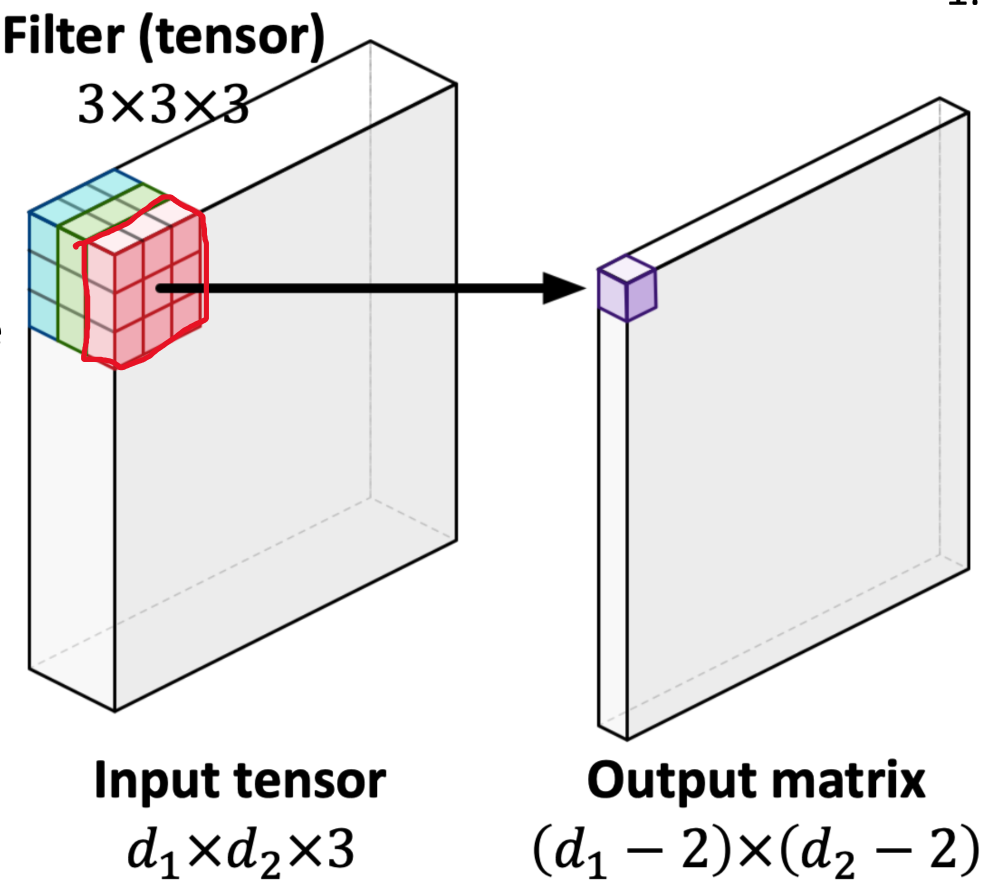


Correlation between data

Q: is there correlation between:

1. locations?

For each channel:
Summarize 9 numbers to one value

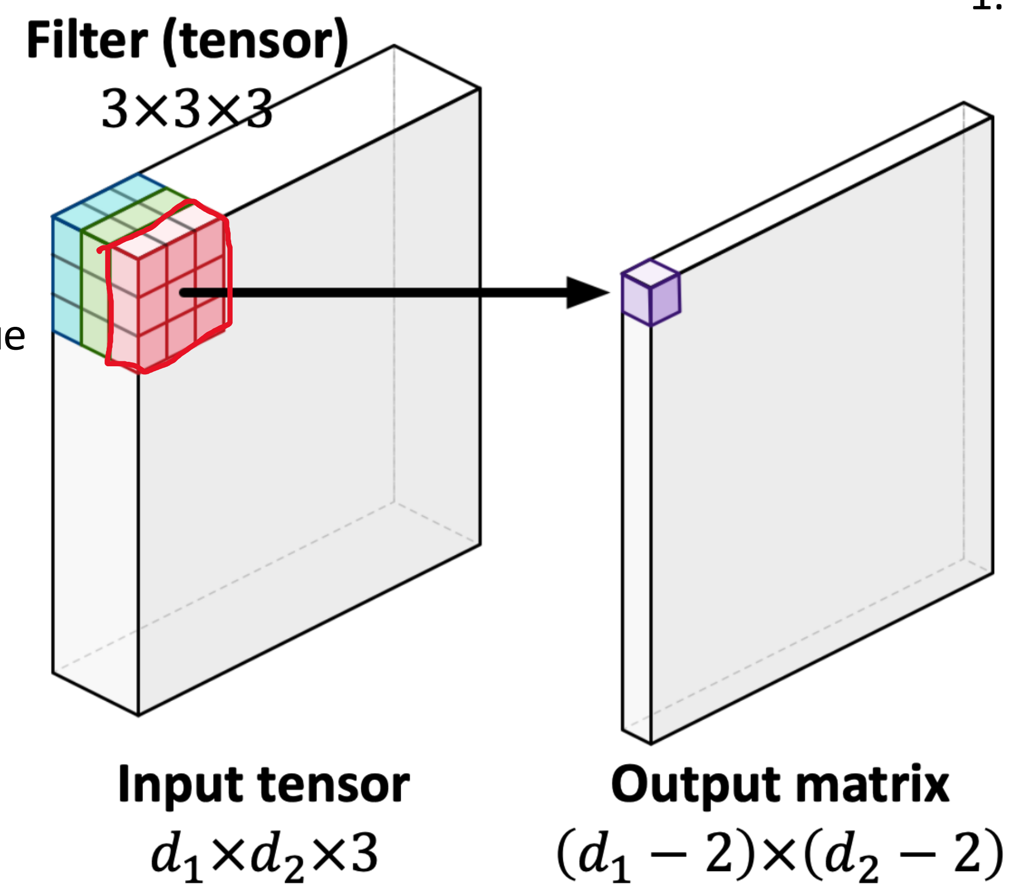


Correlation between data

Q: is there correlation between:

1. locations?

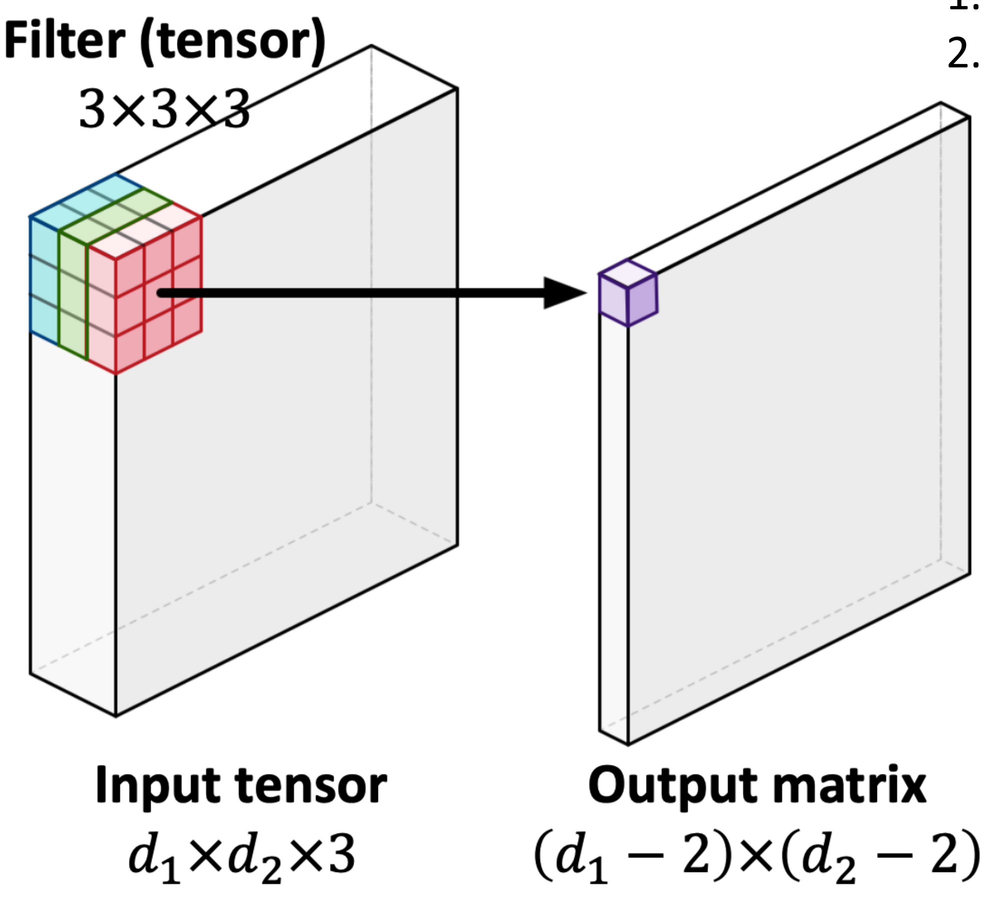
For each channel:
Summarize 9 numbers to one value
(correlation among locations)



Correlation between data

Q: is there correlation between:

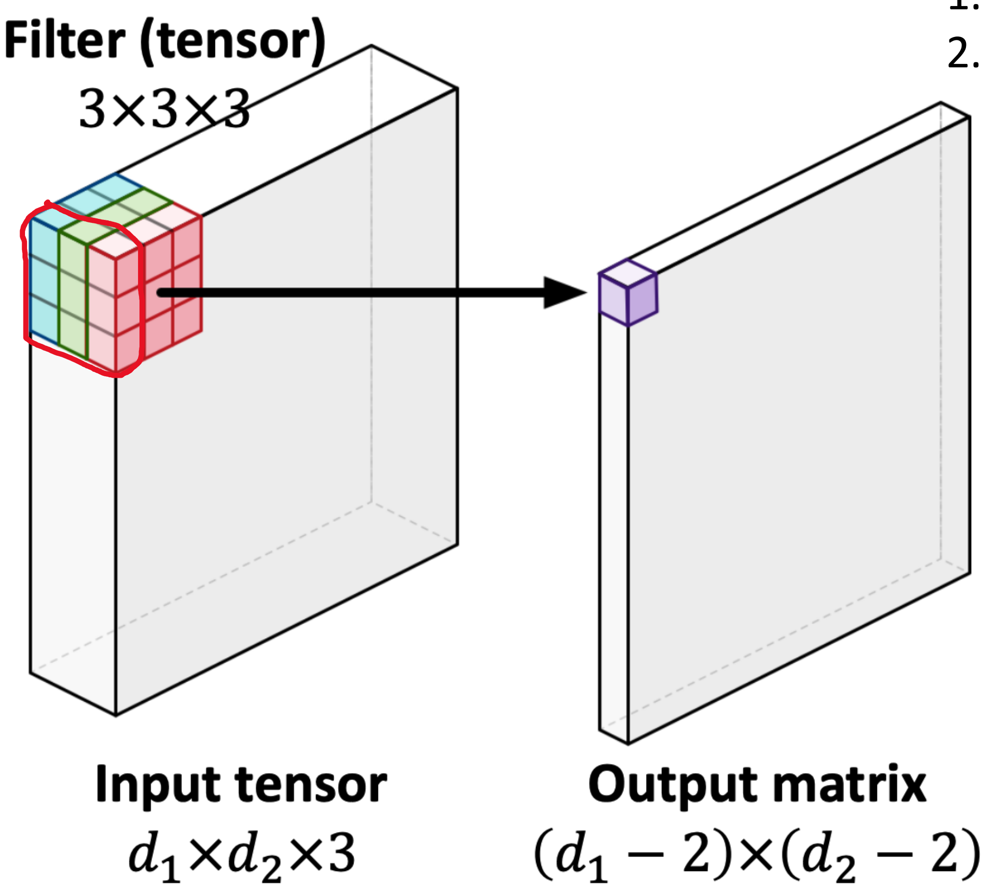
1. locations?
2. channels?



Correlation between data

Q: is there correlation between:

1. locations?
2. channels?

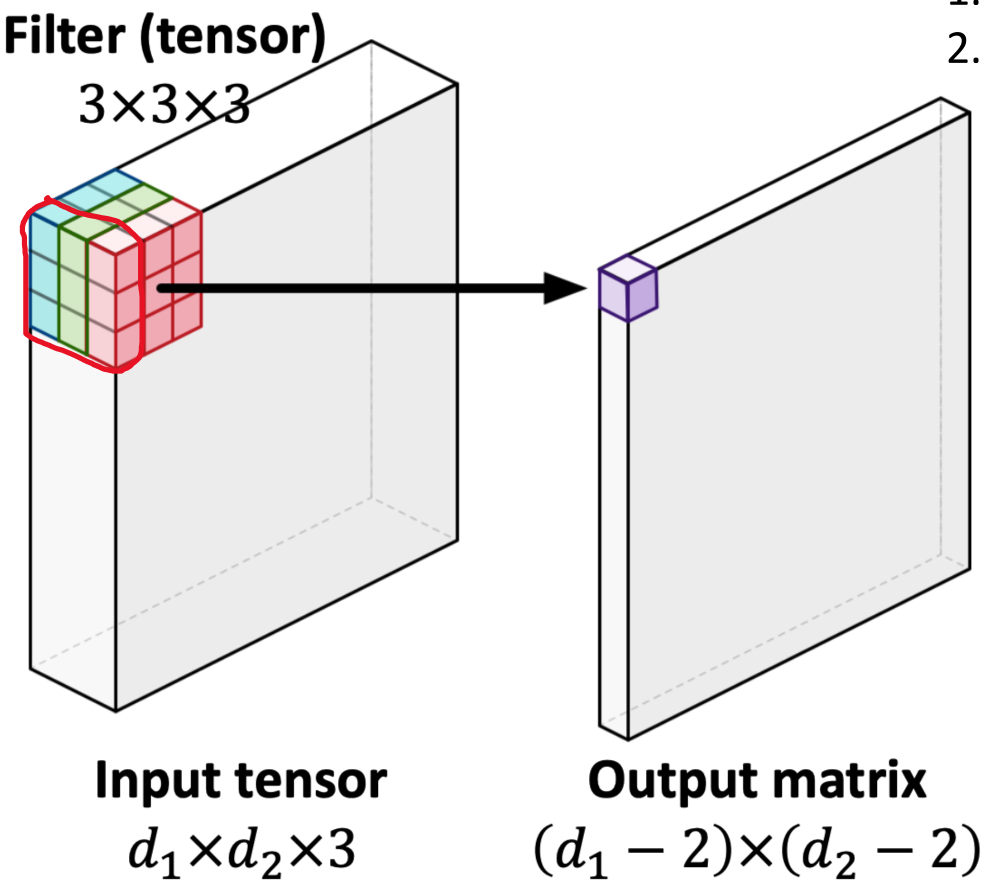


Correlation between data

Q: is there correlation between:

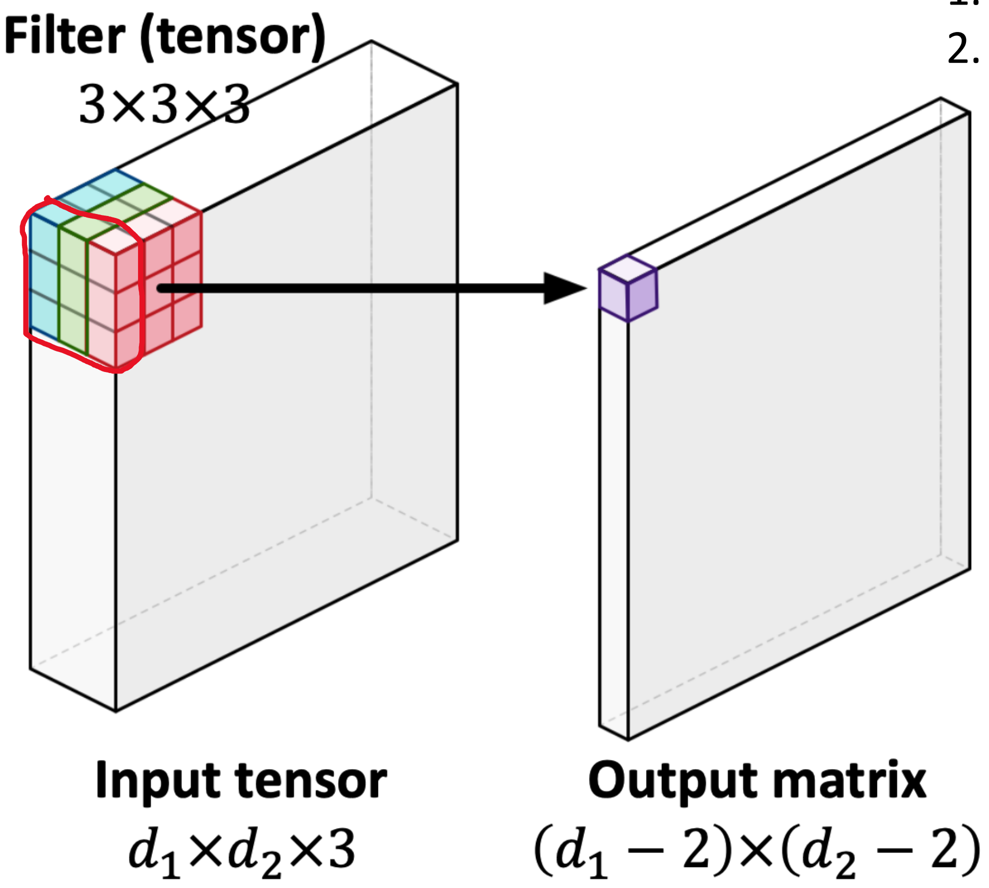
1. locations?
2. channels?

For each location:
Summarize 9 numbers to one value



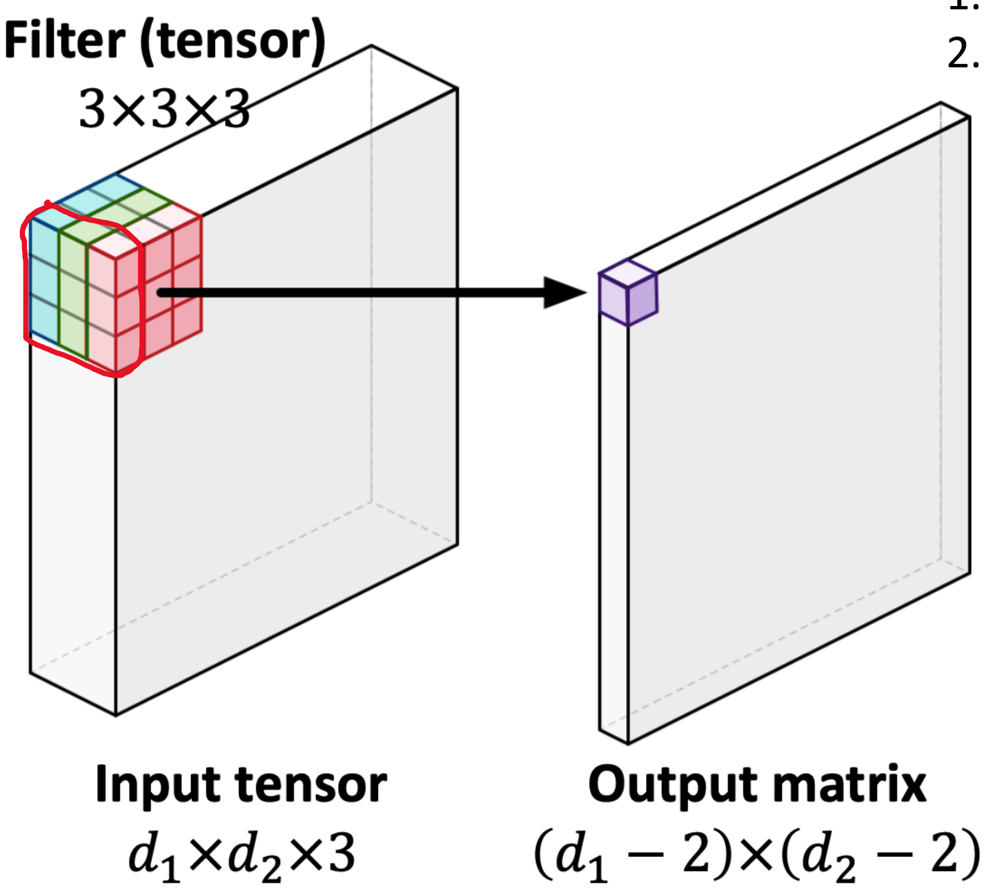
Correlation between data

For each location:
Summarize 9 numbers to one value
(correlation among channels)



Q: is there correlation between:
1. locations?
2. channels?

Correlation between data



Q: is there correlation between:

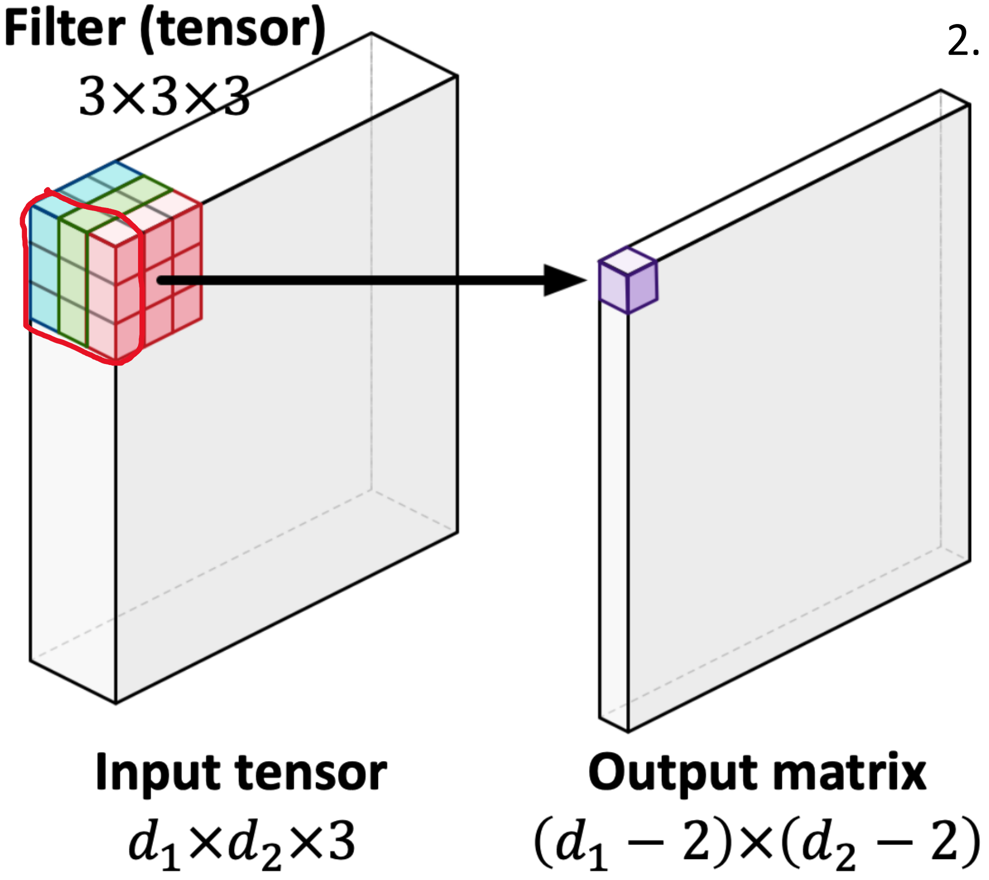
1. locations?
2. channels?

Regard feature maps as a grid

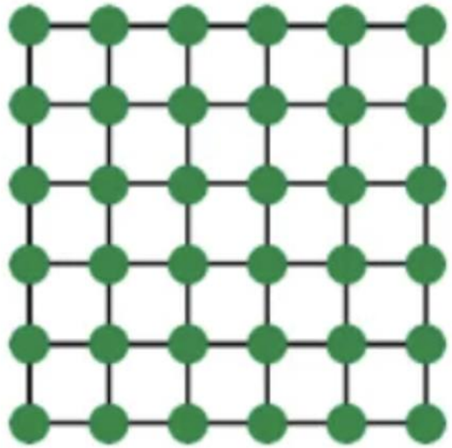
Correlation between data

Q: is there correlation between:

1. locations?
2. channels?

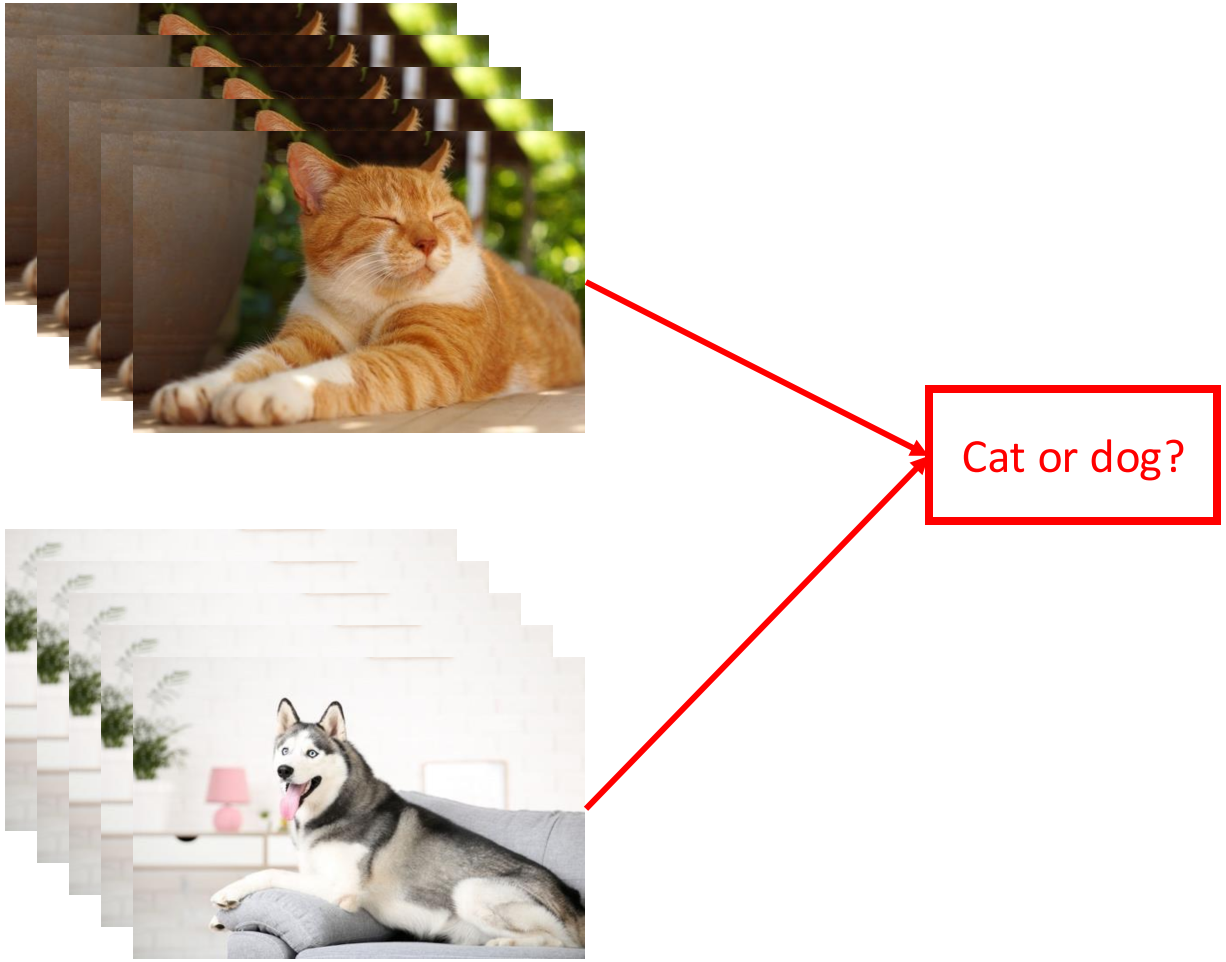


Regard feature maps as a grid



Pixel values or feature values

Correlation between data



Correlation between data

Cat or dog?

Q: is there correlation between images?

Correlation between data

Cat or dog?

Q: is there correlation between images?
No

Correlation between data

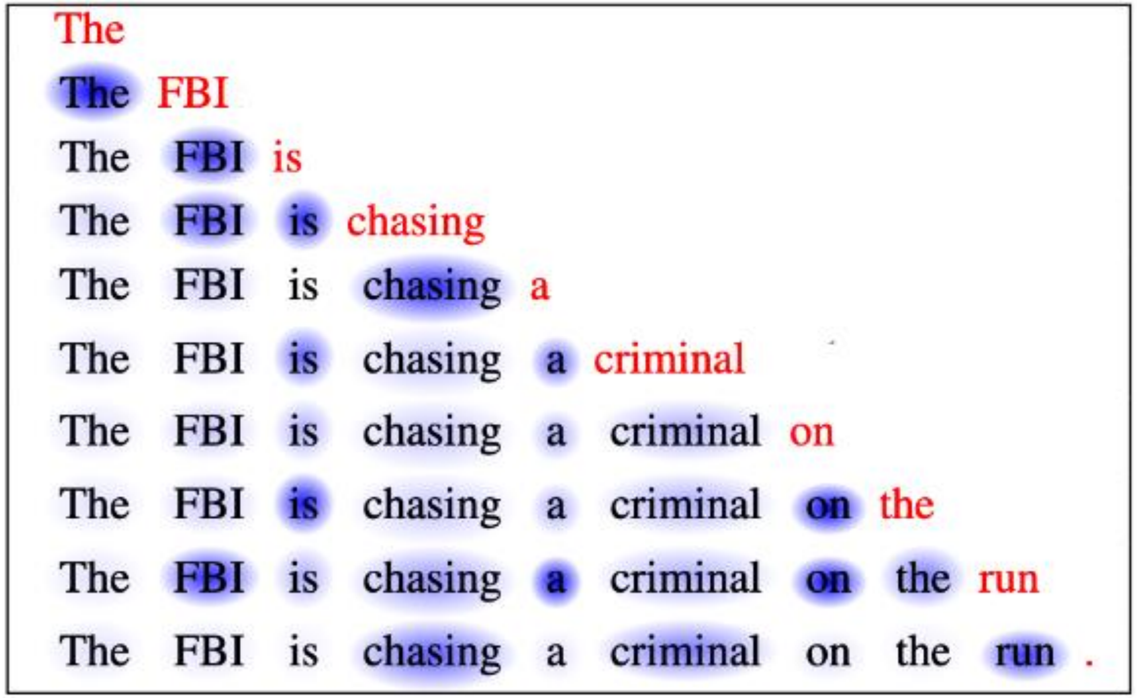
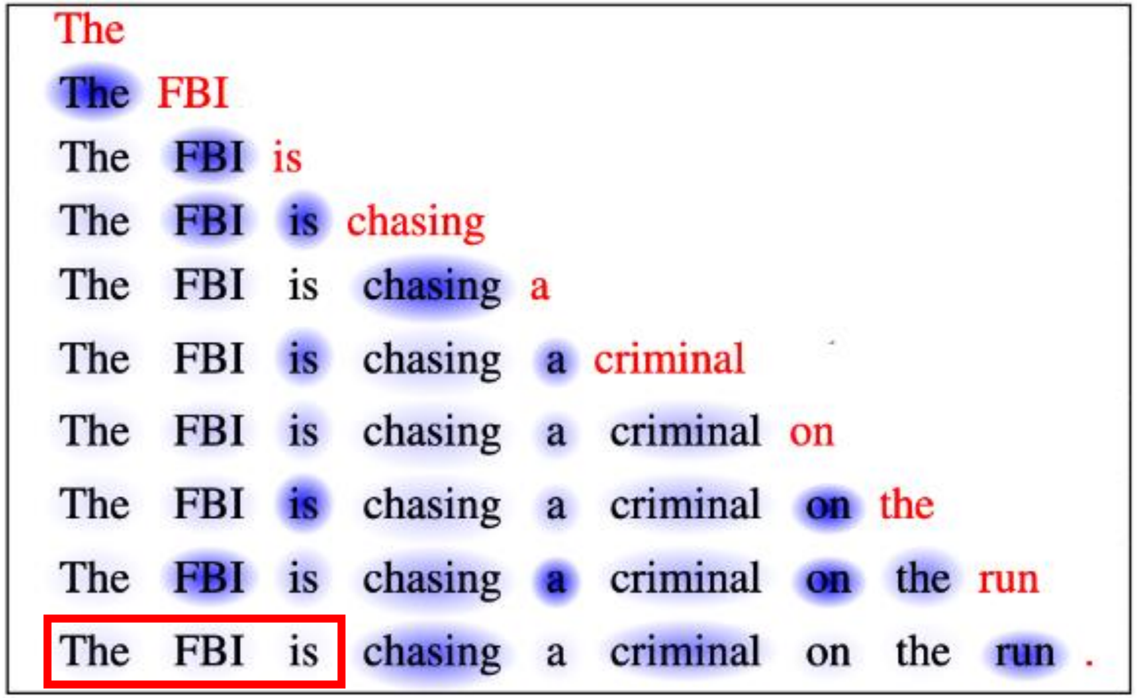


Figure is from the paper " Long Short-Term Memory-Networks for Machine Reading."

Correlation between data



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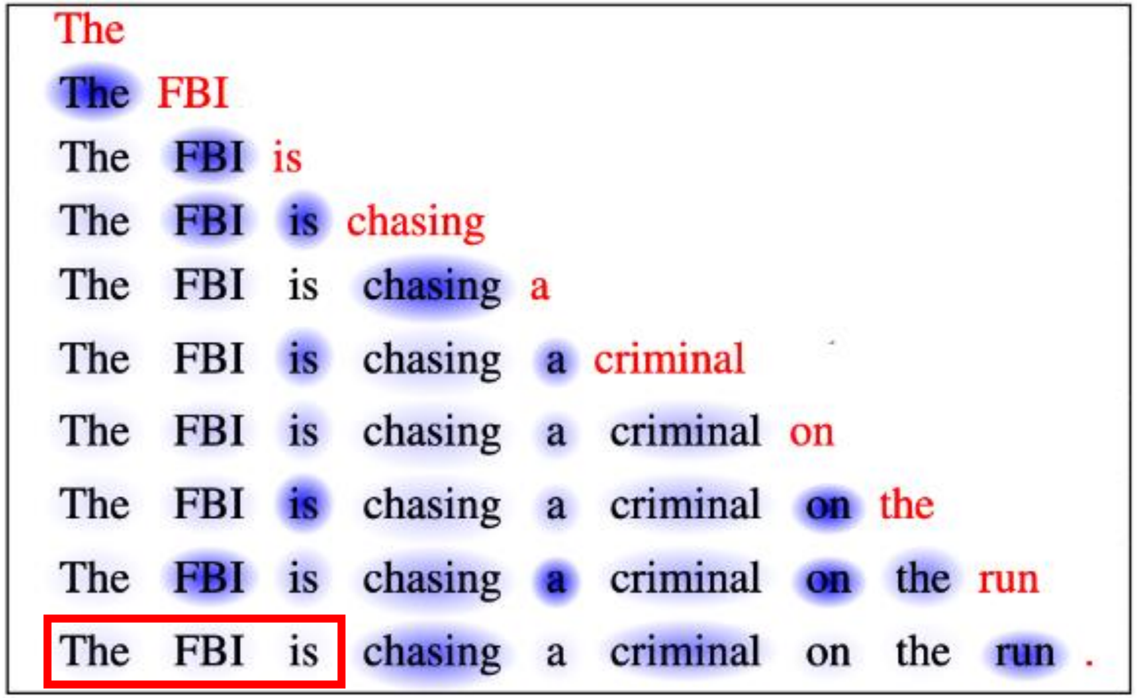

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The FBI is

Correlation between data

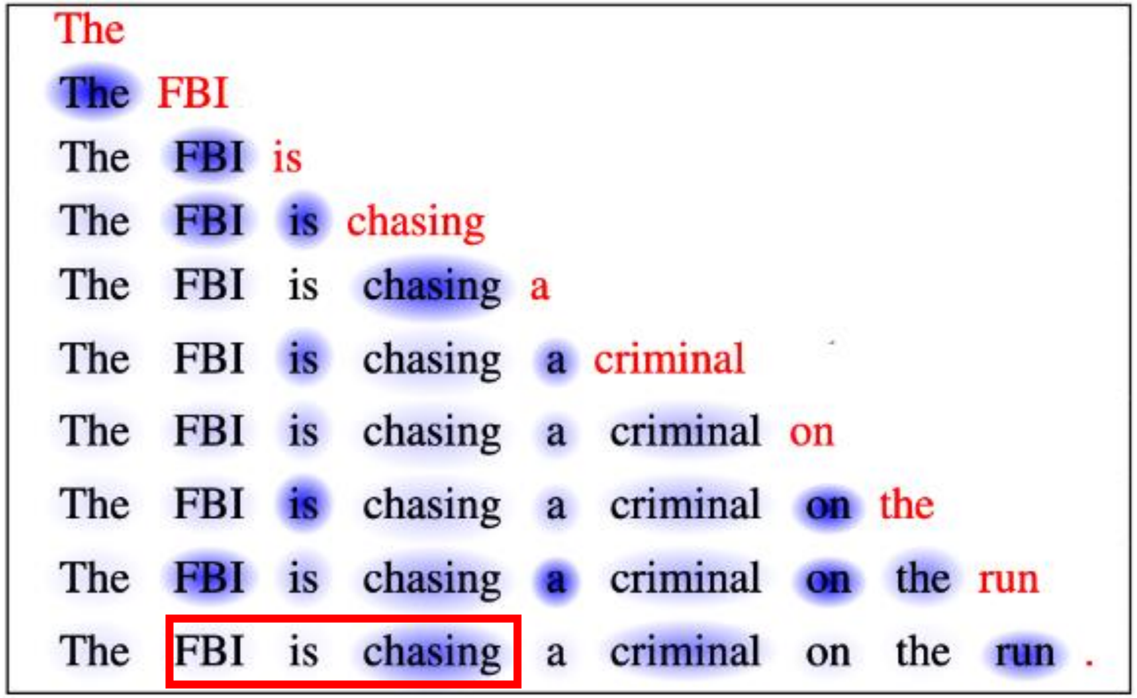
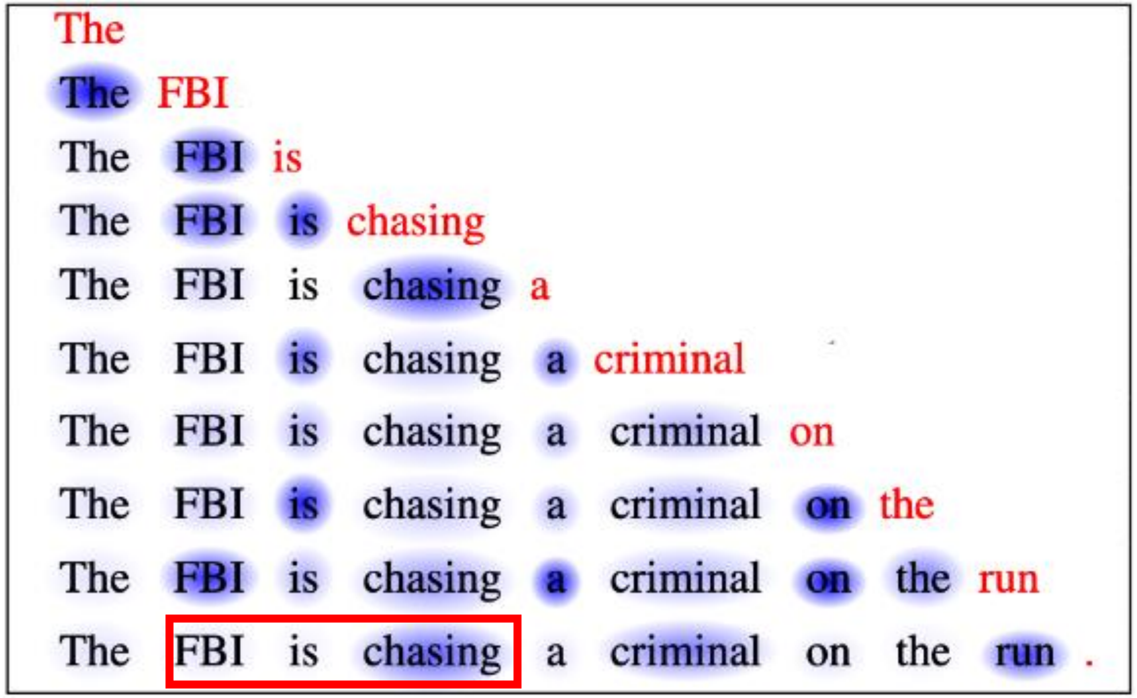


Figure is from the paper " Long Short-Term Memory-Networks for Machine Reading."

The FBI is

FBI is chasings

Correlation between data



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The FBI is → FBI is chasings

Correlation between data

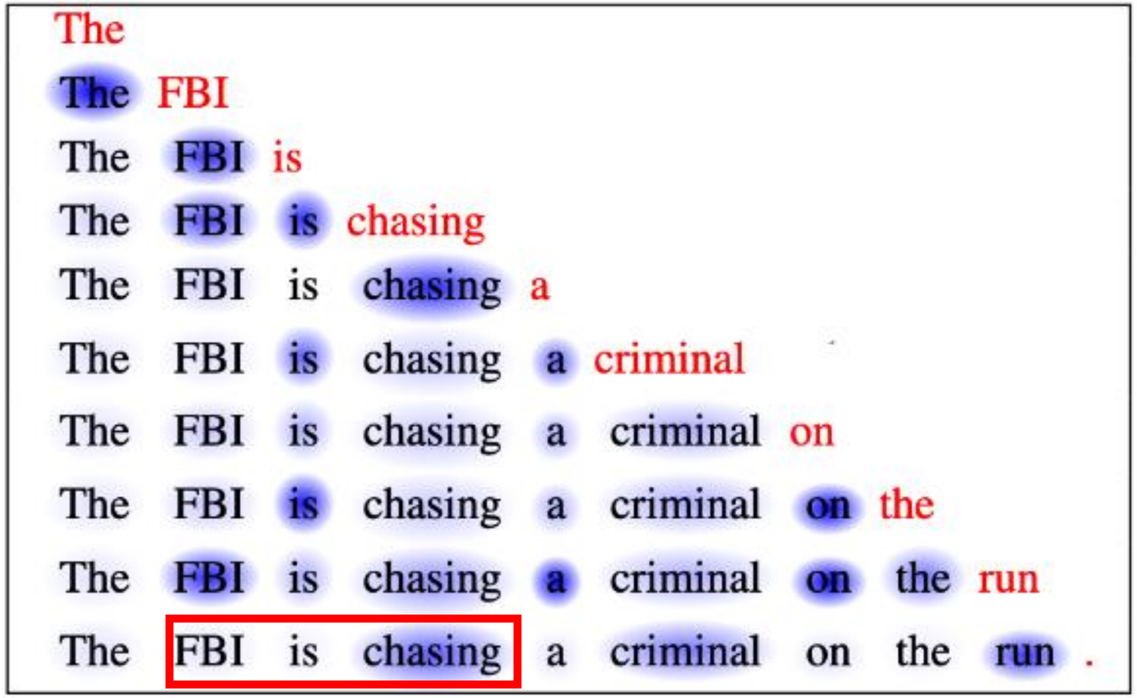


Figure is from the paper " Long Short-Term Memory-Networks for Machine Reading."

The FBI is → FBI is chasings

Sequence data 

Correlation between data

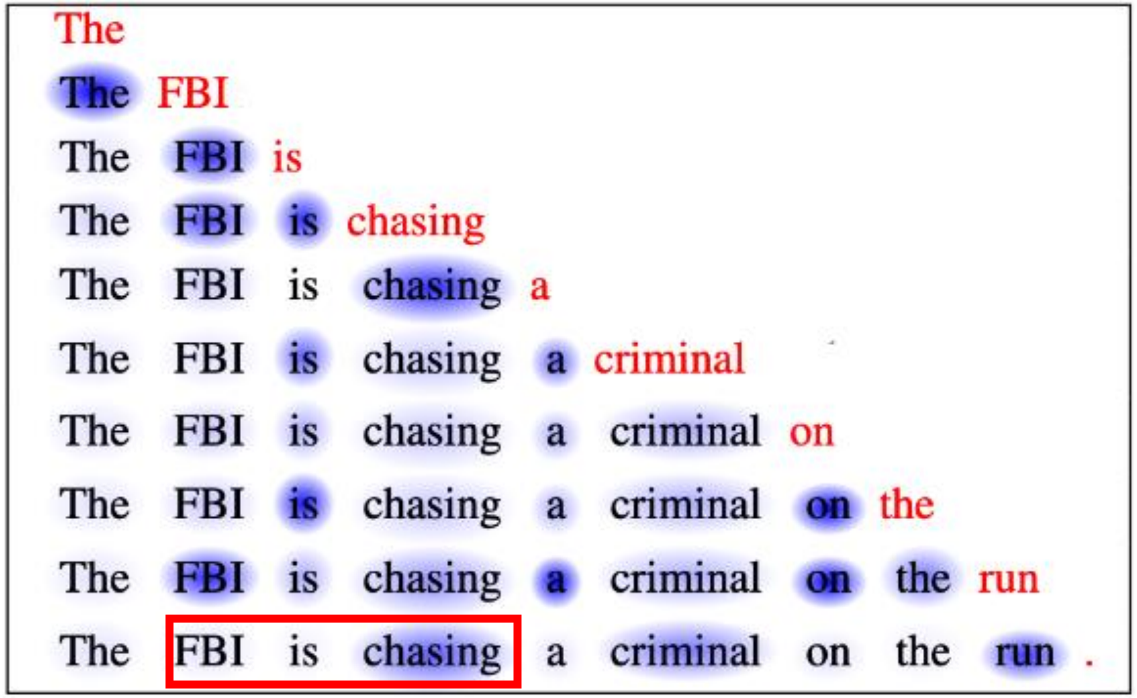


Figure is from the paper " Long Short-Term Memory-Networks for Machine Reading."

The FBI is → FBI is chasings

Sequence data 

Q: what if we need more complicated correlation?

Correlation between data

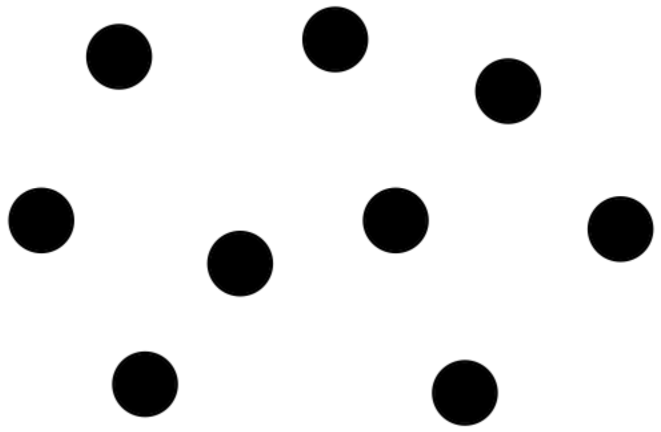
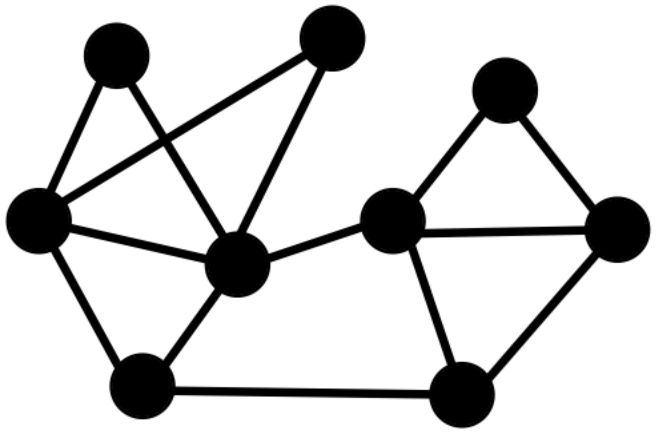


Image credit http://web.stanford.edu/class/cs224w/slides/01-intro.pdf

Correlation between data



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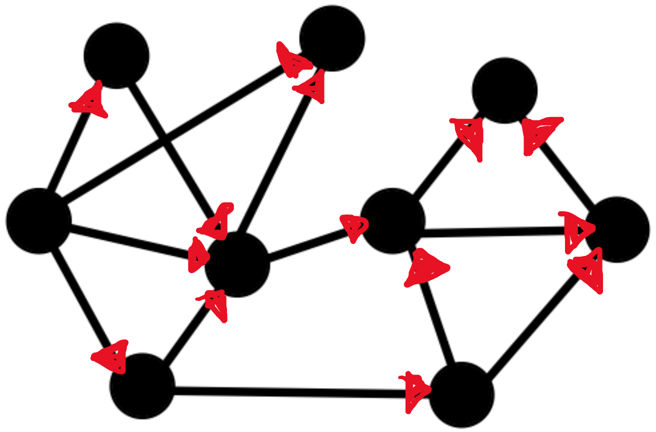
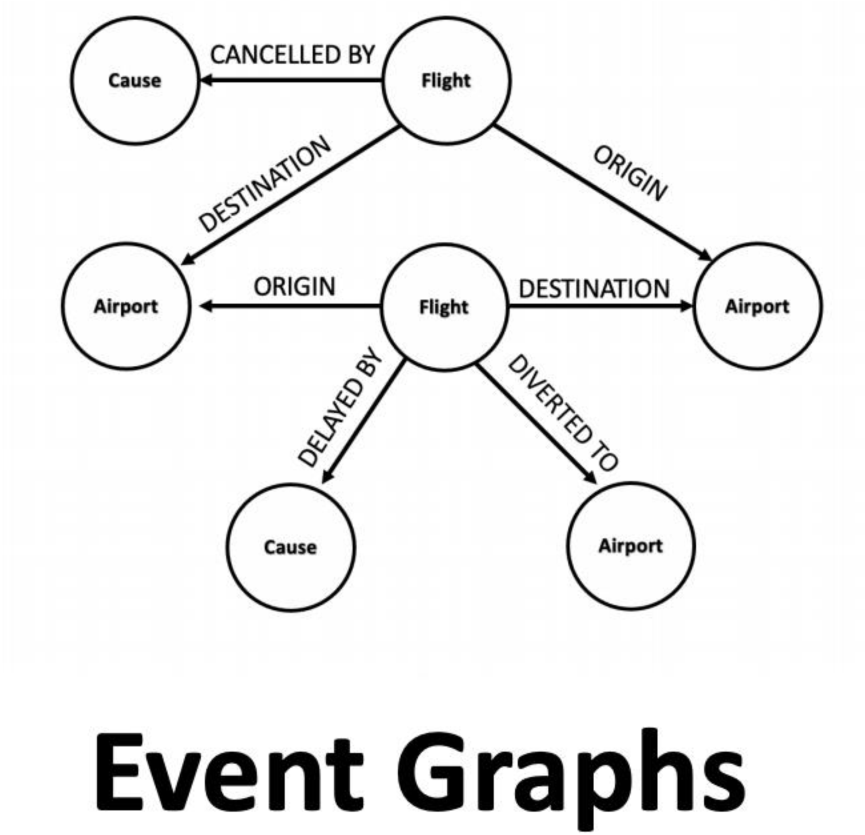
Correlation between data



(with directions)

Image credit http://web.stanford.edu/class/cs224w/slides/01-intro.pdf

Correlation between data



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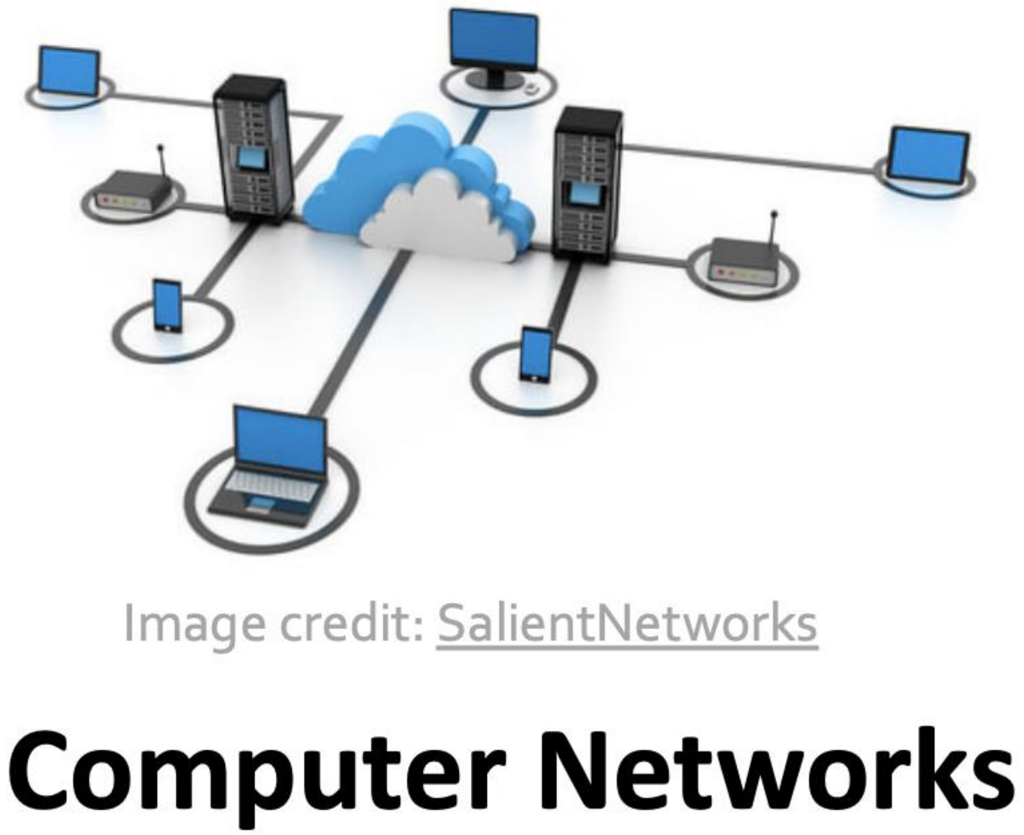

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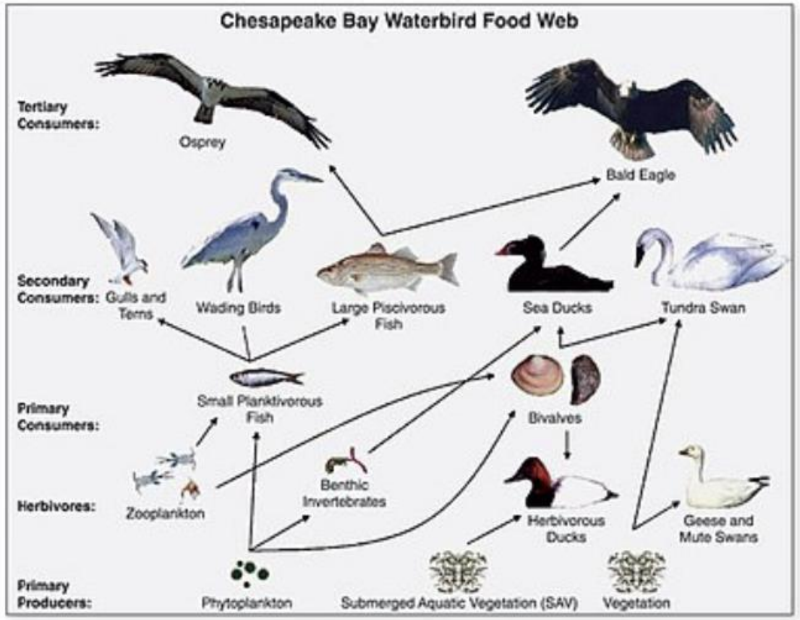


Image credit: Wikipedia

Food Webs

Image credit http://web.stanford.edu/class/cs224w/slides/01-intro.pdf

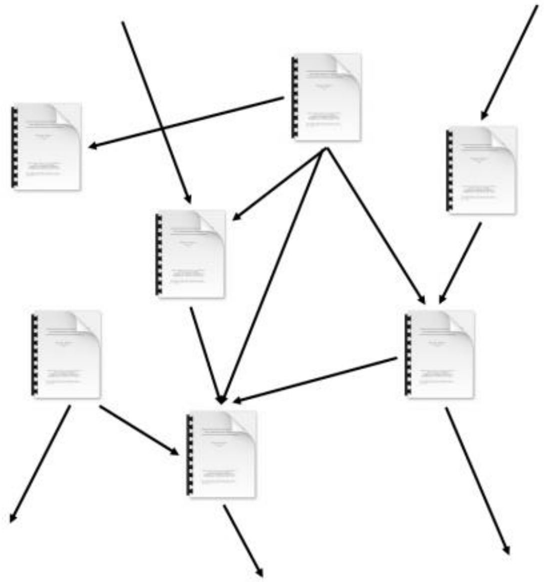
Correlation between data

Image credit: Medium

Social Networks

Image credit http://web.stanford.edu/class/cs224w/slides/01-intro.pdf

Correlation between data



Citation Networks

Image credit http://web.stanford.edu/class/cs224w/slides/01-intro.pdf

Correlation between data

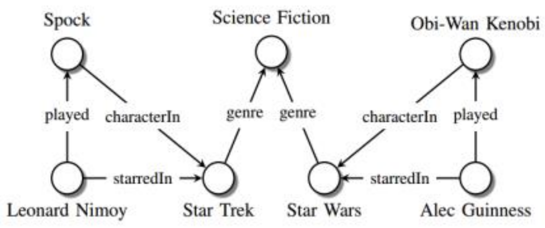


Image credit: Maximilian Nickel et al

Knowledge Graphs

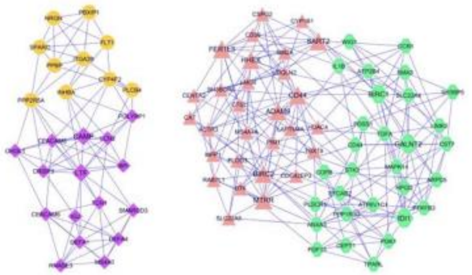


Image credit: ese.wustl.edu

Regulatory Networks

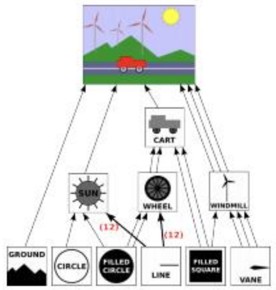


Image credit: math.hws.edu

Scene Graphs

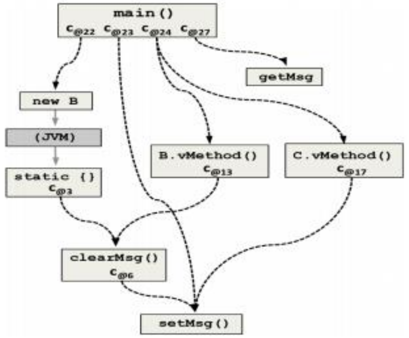


Image credit: ResearchGate

Code Graphs

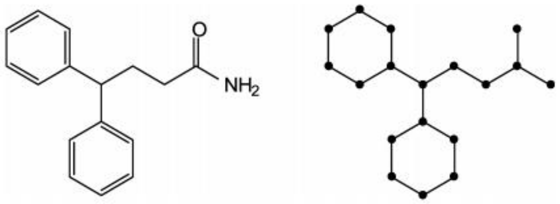


Image credit: MDPI

Molecules

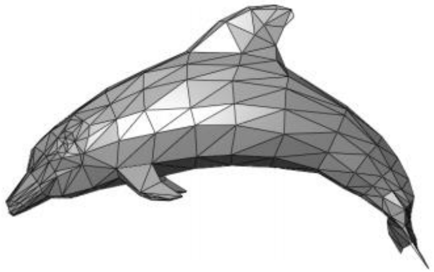
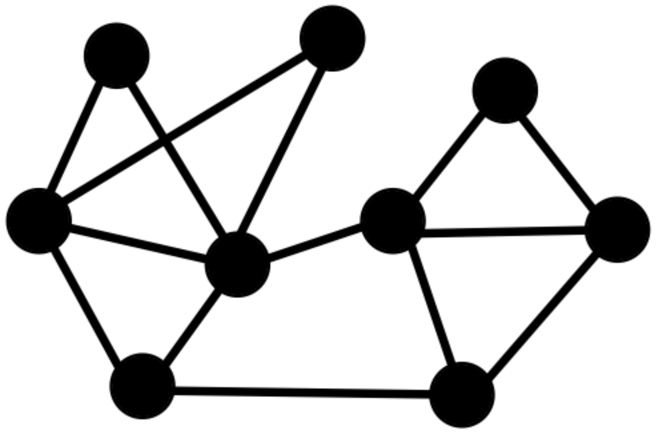


Image credit: Wikipedia

3D Shapes

Image credit http://web.stanford.edu/class/cs224w/slides/01-intro.pdf

Correlation between data



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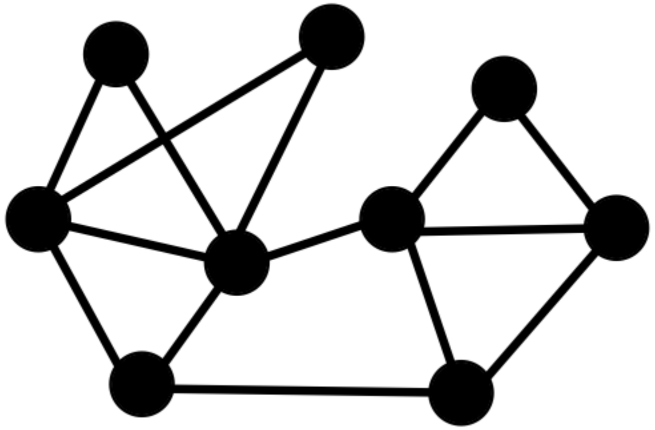
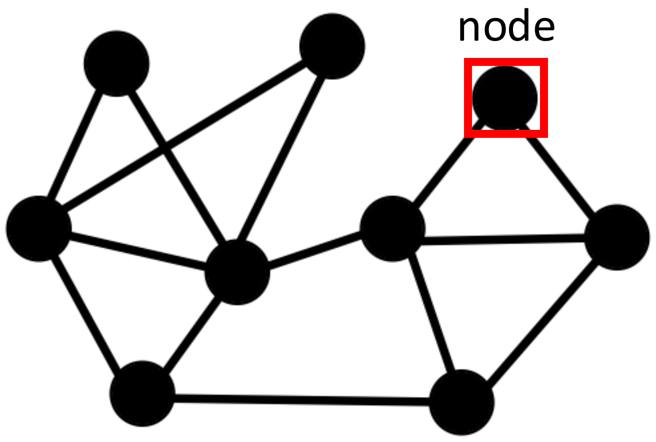
Correlation between data



undirected graph

Image credit http://web.stanford.edu/class/cs224w/slides/01-intro.pdf

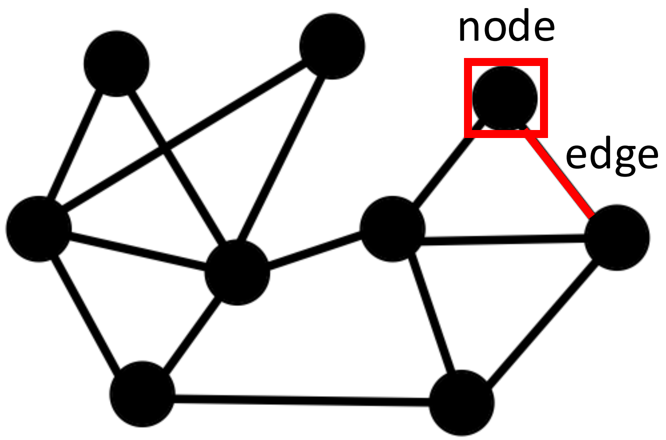
Correlation between data



undirected graph

Image credit http://web.stanford.edu/class/cs224w/slides/01-intro.pdf

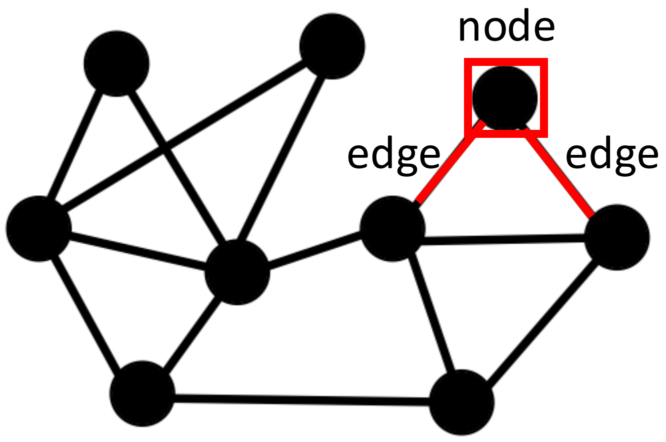
Correlation between data



undirected graph

Image credit http://web.stanford.edu/class/cs224w/slides/01-intro.pdf

Correlation between data



undirected graph

Image credit http://web.stanford.edu/class/cs224w/slides/01-intro.pdf

Correlation between data

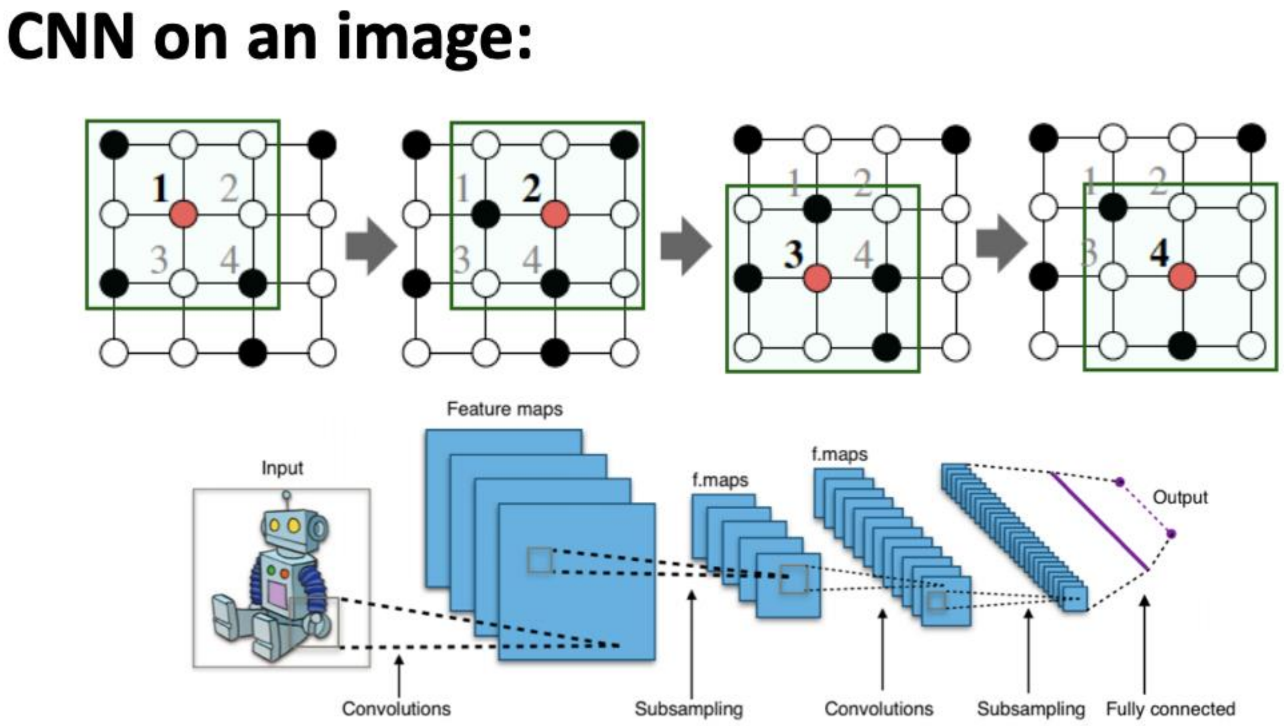


Image credit https://web.stanford.edu/class/cs224w/slides/08-GNN.pdf

Machine learning tasks for graph data

- Node level
- Edge level
- Community level
- Graph level

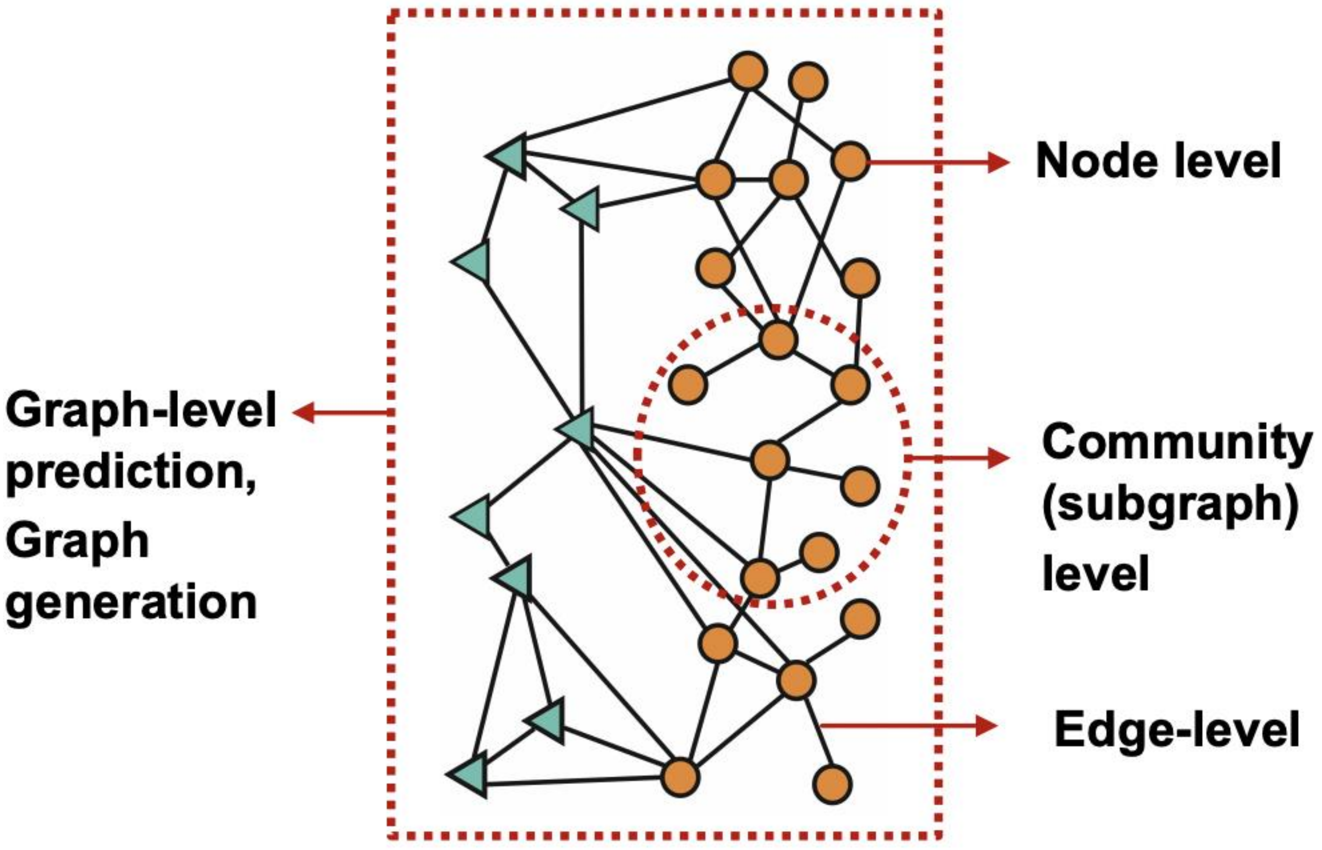


Image credit http://web.stanford.edu/class/cs224w/slides/01-intro.pdf

Machine learning tasks for graph data

- Node level
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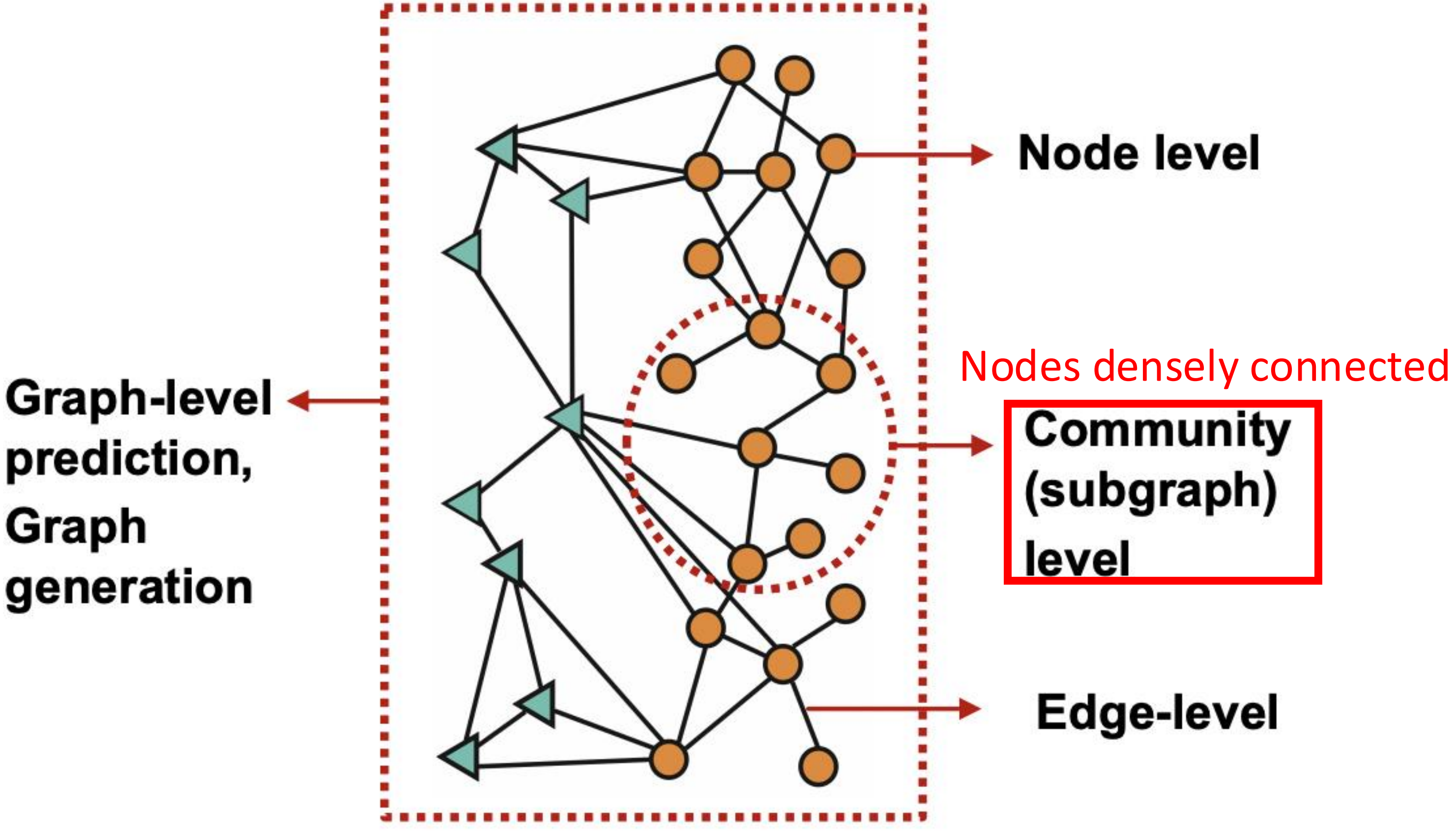


Image credit http://web.stanford.edu/class/cs224w/slides/01-intro.pdf

Machine learning tasks for graph data

classification

- Node level
- Edge level
- Community level
- Graph level

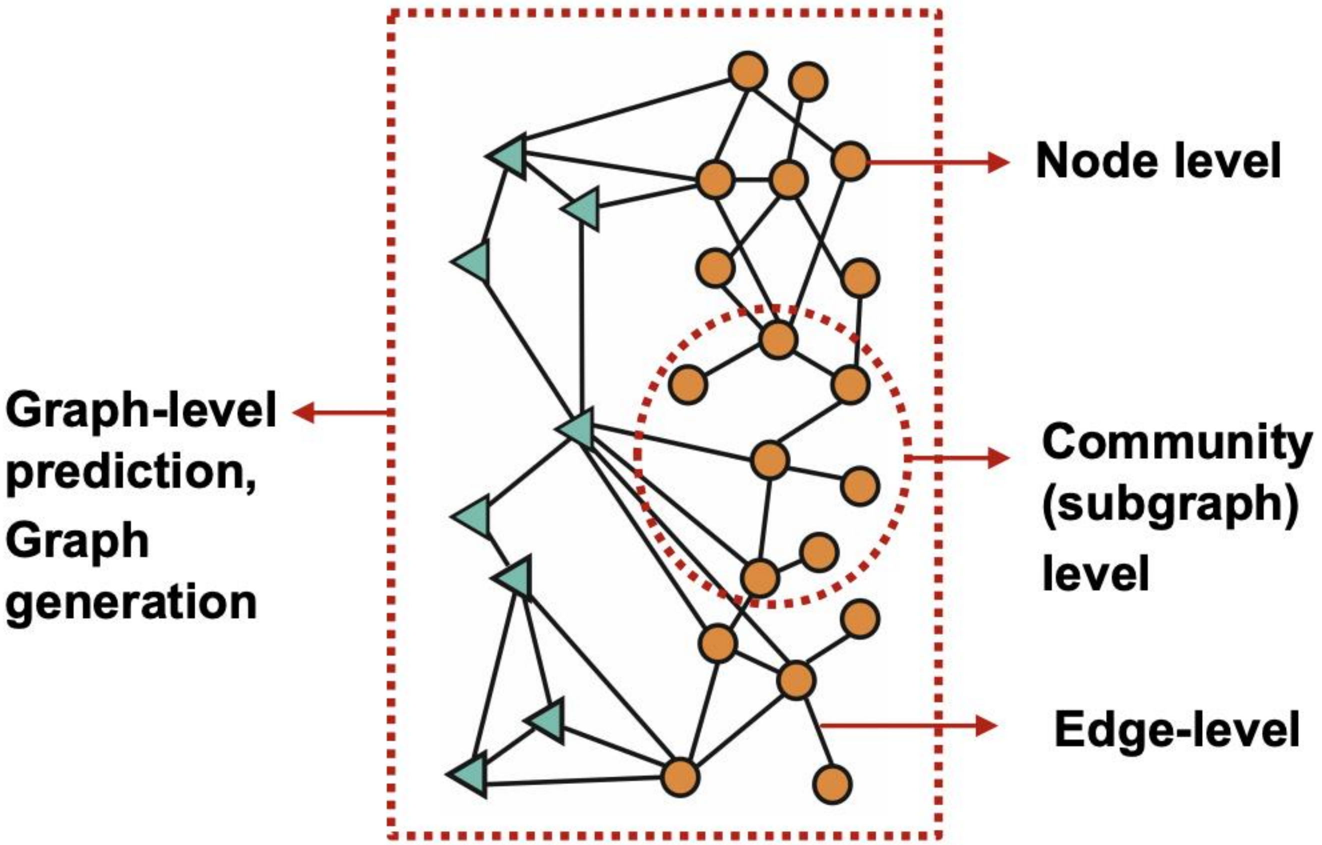


Image credit http://web.stanford.edu/class/cs224w/slides/01-intro.pdf

Social network

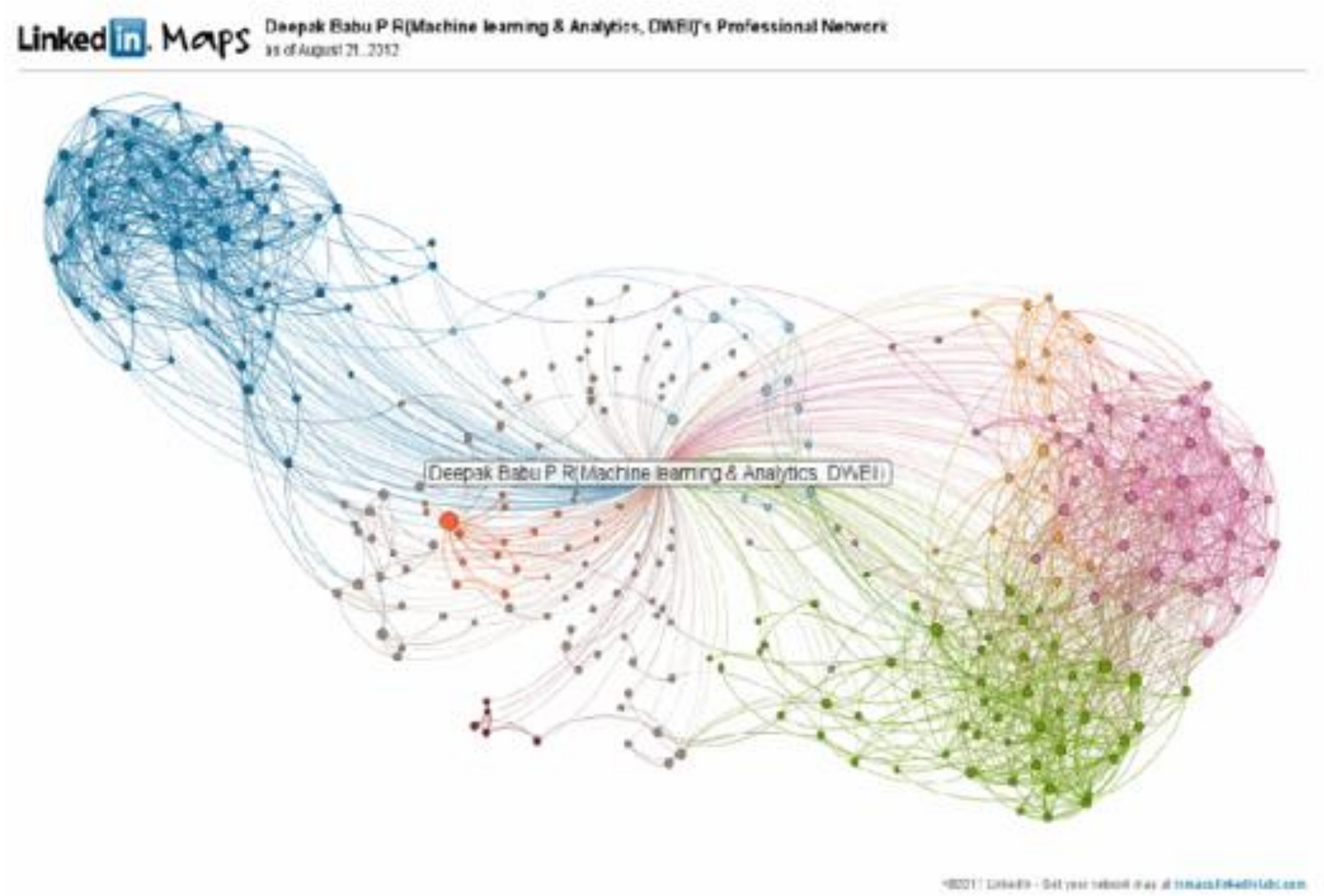
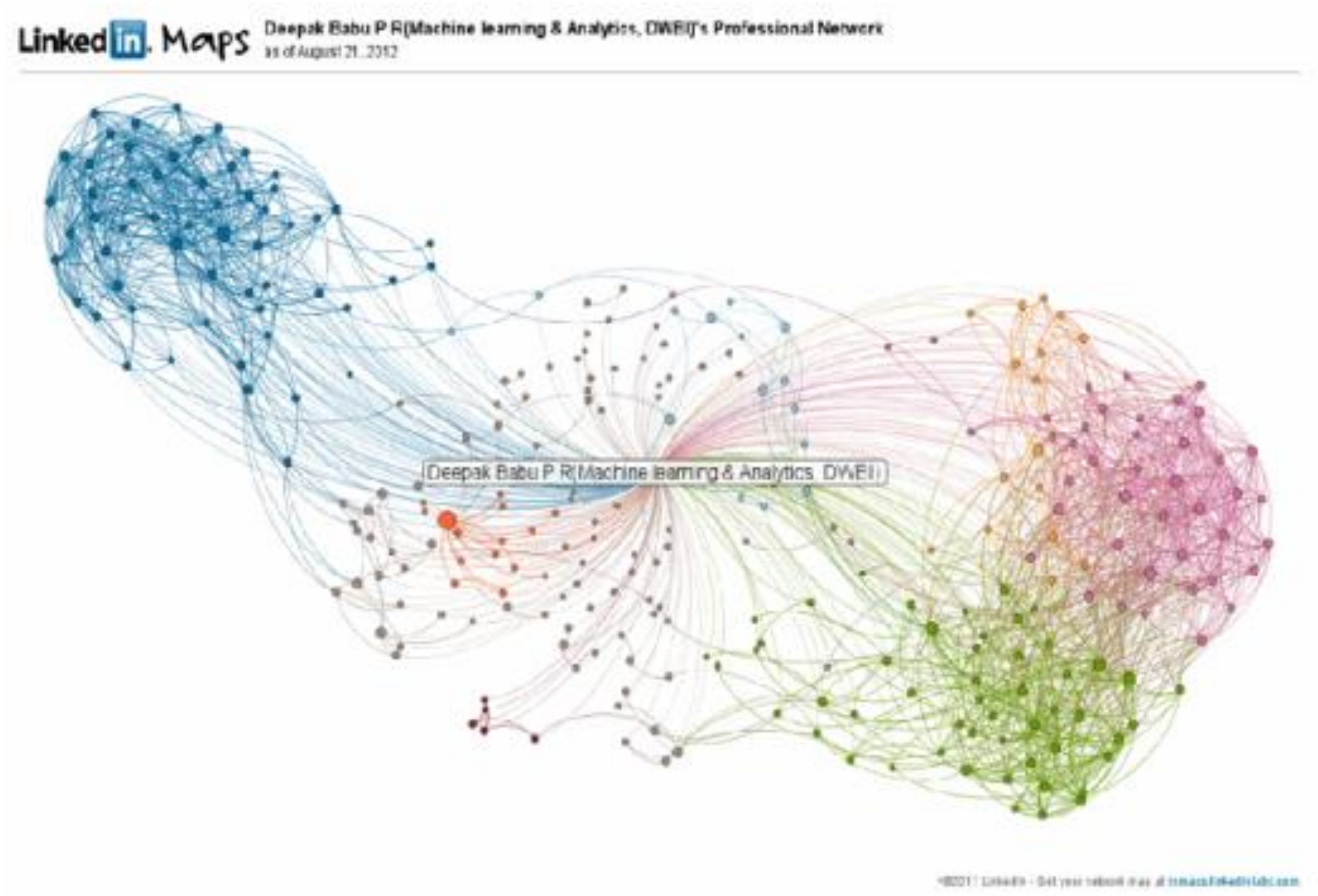


Image credit https://prdeepakbabu.wordpress.com/2012/10/06/social-network-analytics-visualization/

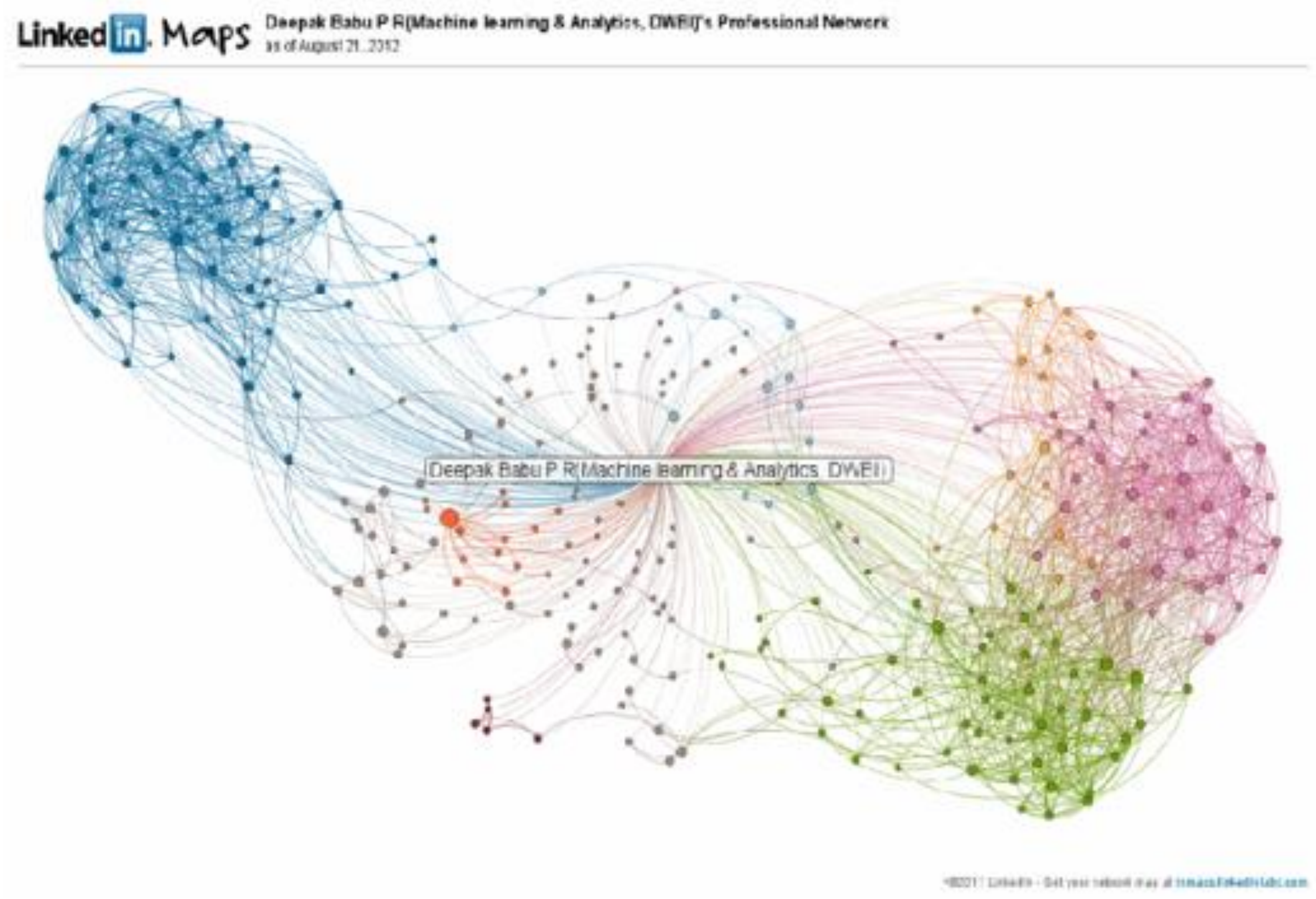
Social network



Nodes: users

Image credit https://prdeepakbabu.wordpress.com/2012/10/06/social-network-analytics-visualization/

Social network

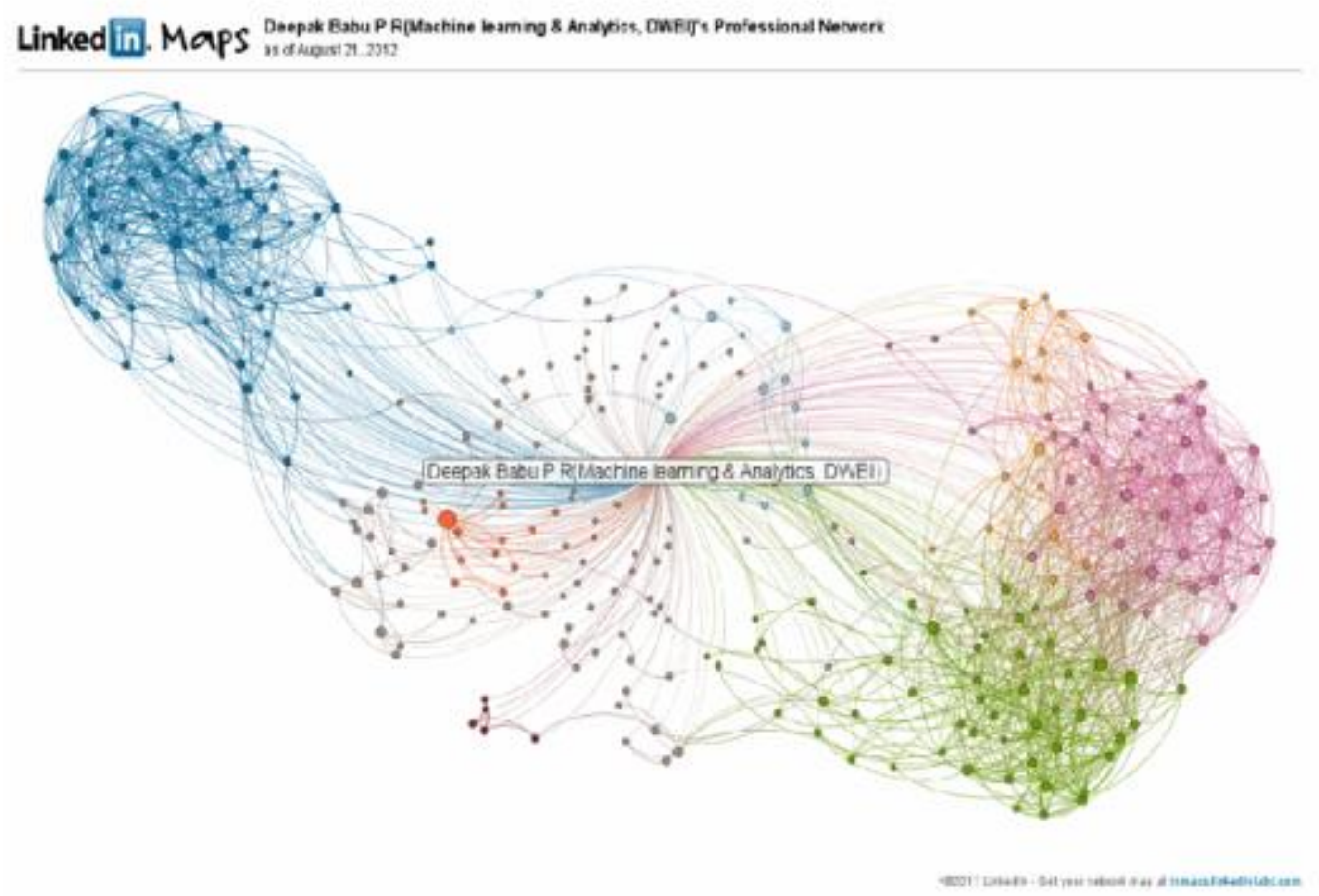


Nodes: users

Edges: interactions

Image credit https://prdeepakbabu.wordpress.com/2012/10/06/social-network-analytics-visualization/

Social network



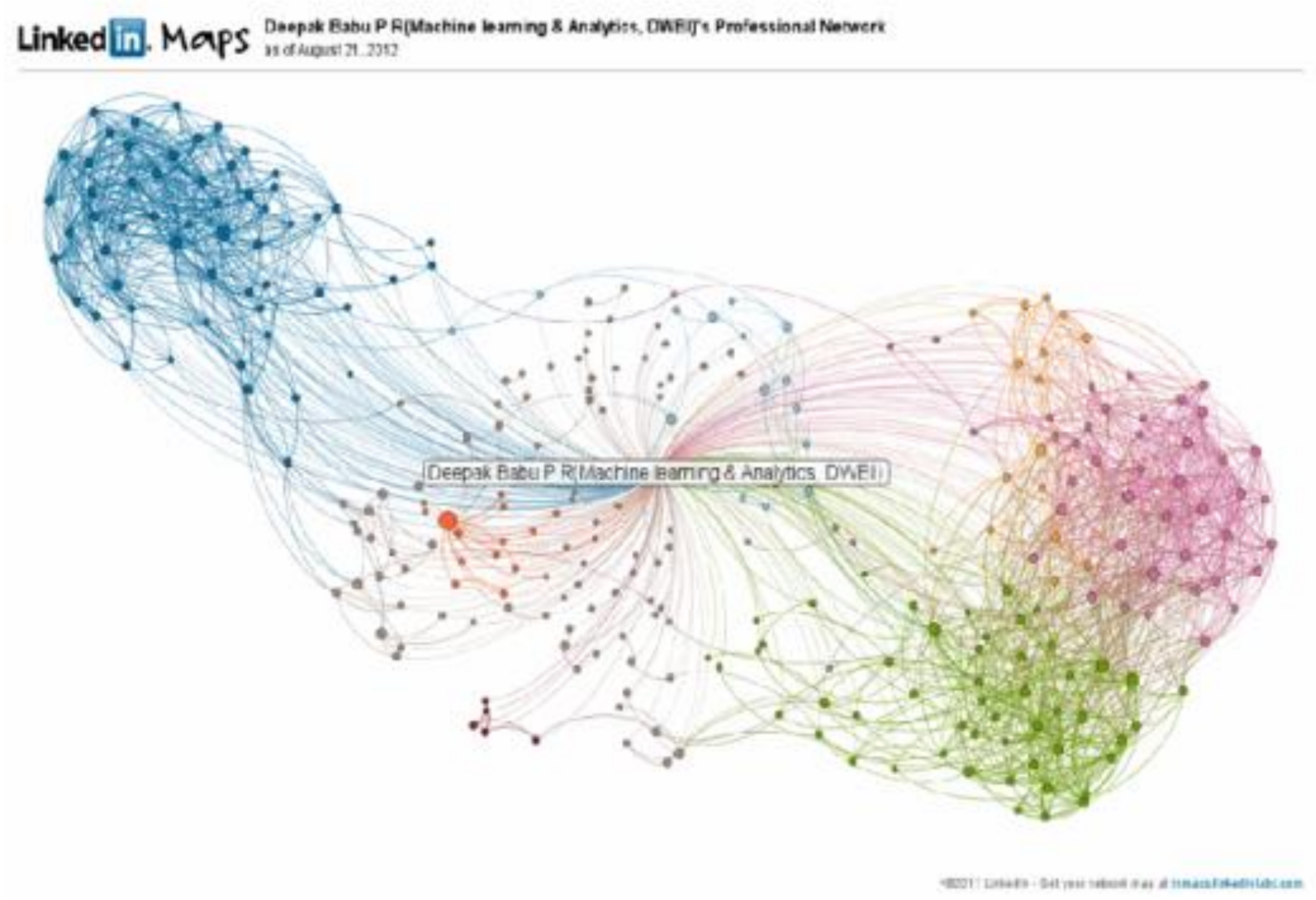
Nodes: users

Edges: interactions

FB: add friend

Image credit https://prdeepakbabu.wordpress.com/2012/10/06/social-network-analytics-visualization/

Social network



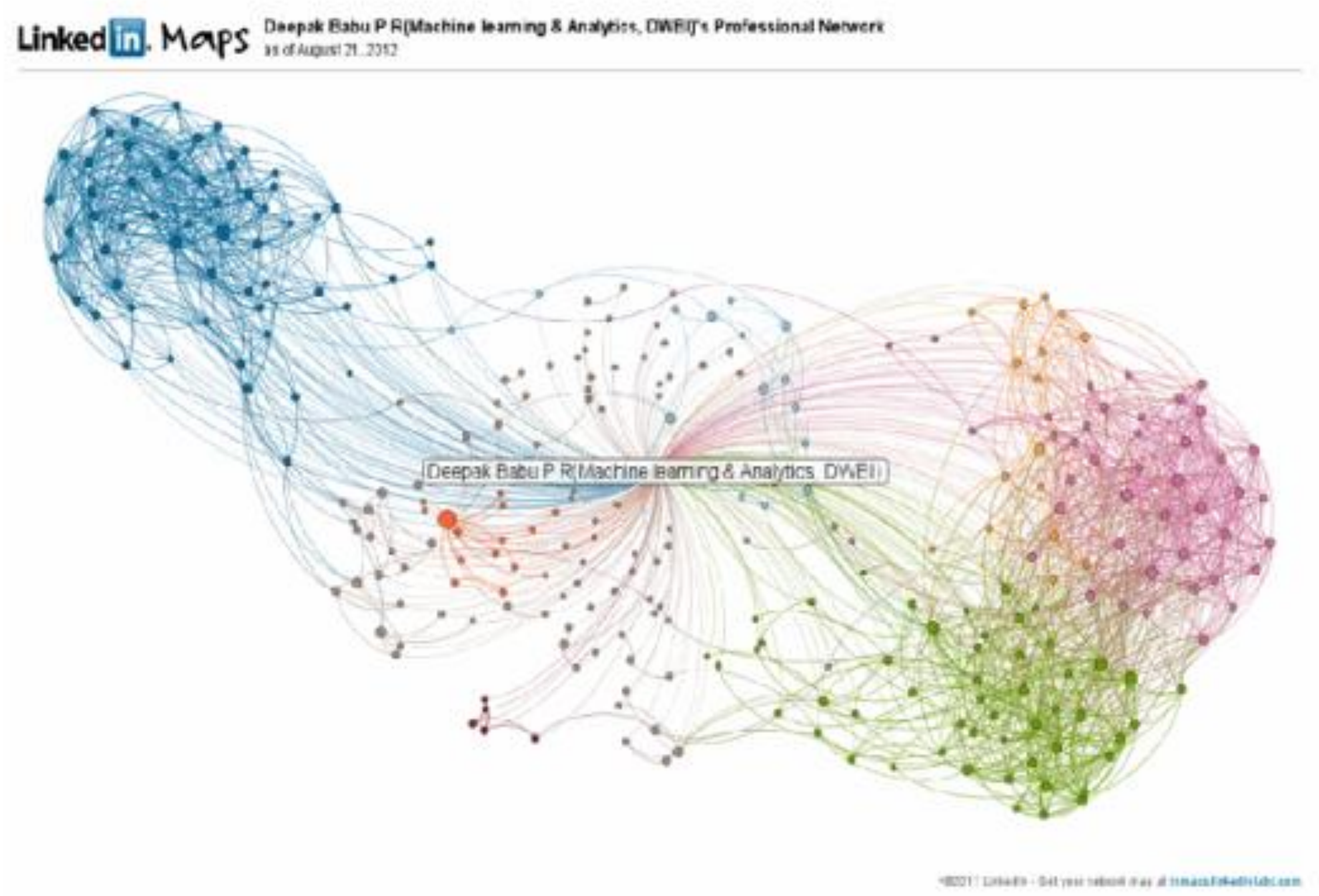
Nodes: users

Edges: interactions

LinkedIn: connect

Image credit https://prdeepakbabu.wordpress.com/2012/10/06/social-network-analytics-visualization/

Social network



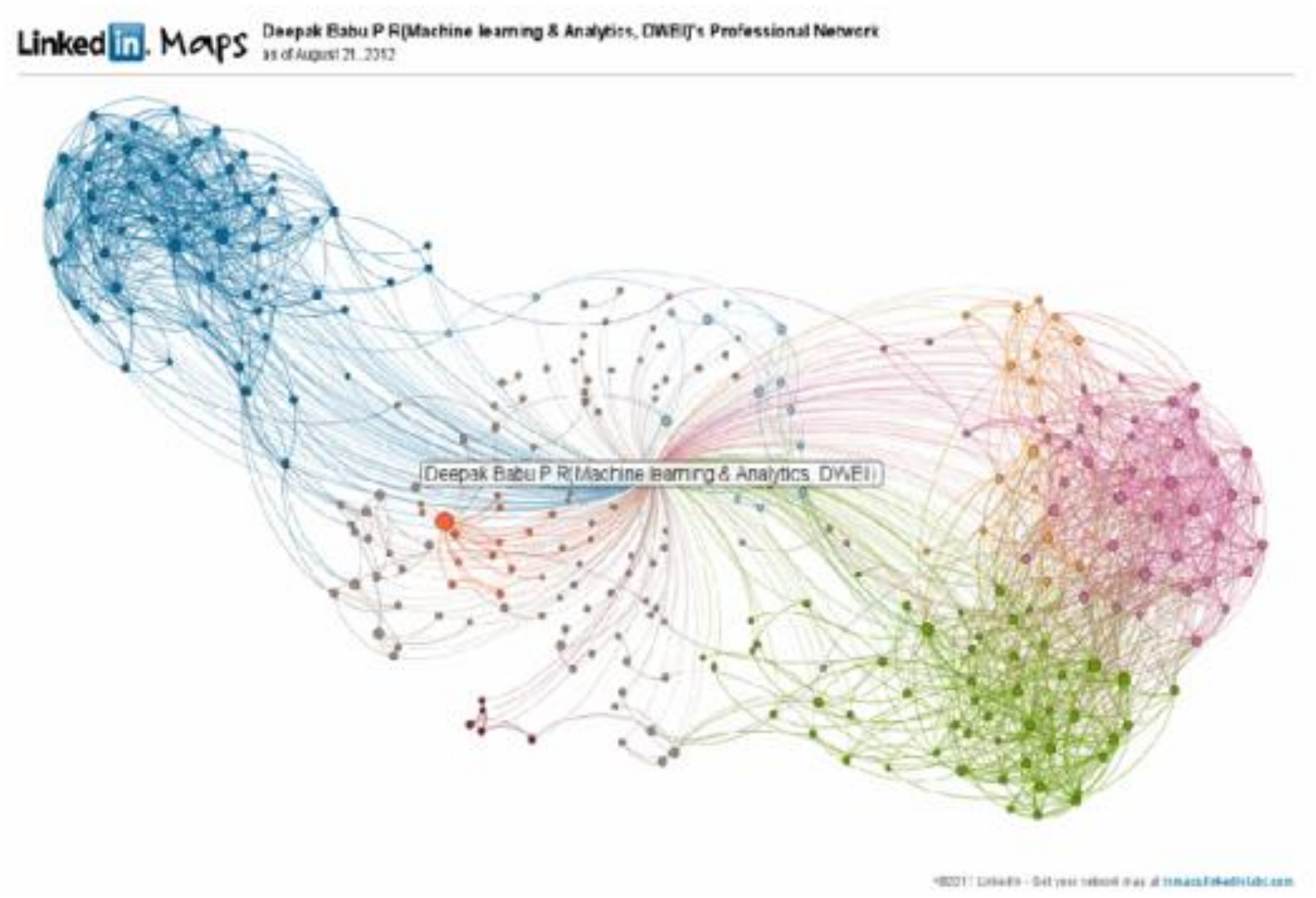
Nodes: users

Edges: interactions

Amazon: same purchase

Image credit https://prdeepakbabu.wordpress.com/2012/10/06/social-network-analytics-visualization/

Social network



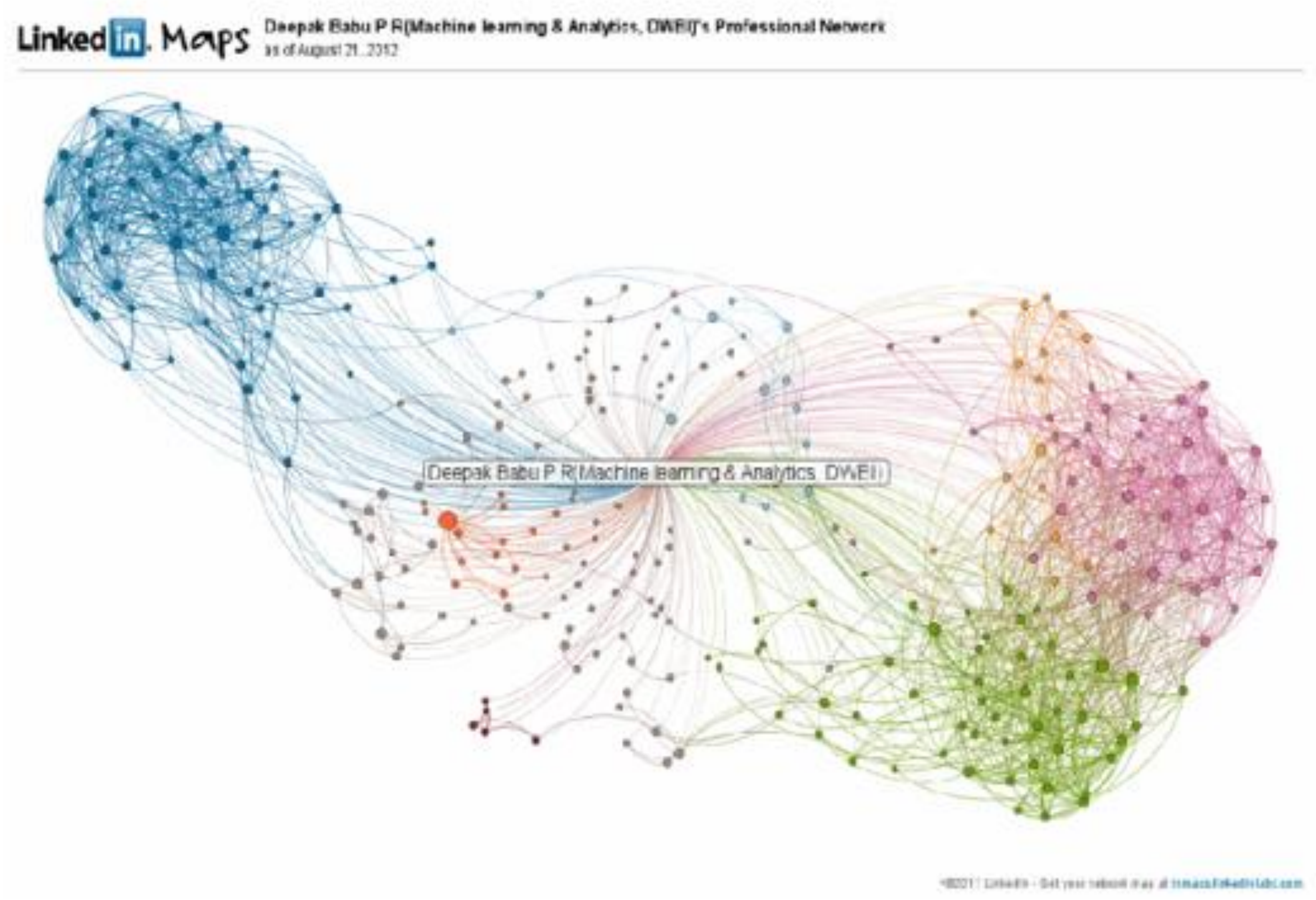
Nodes: users

Edges: interactions

Node classification:
Group nodes by their properties

Image credit https://prdeepakbabu.wordpress.com/2012/10/06/social-network-analytics-visualization/

Social network



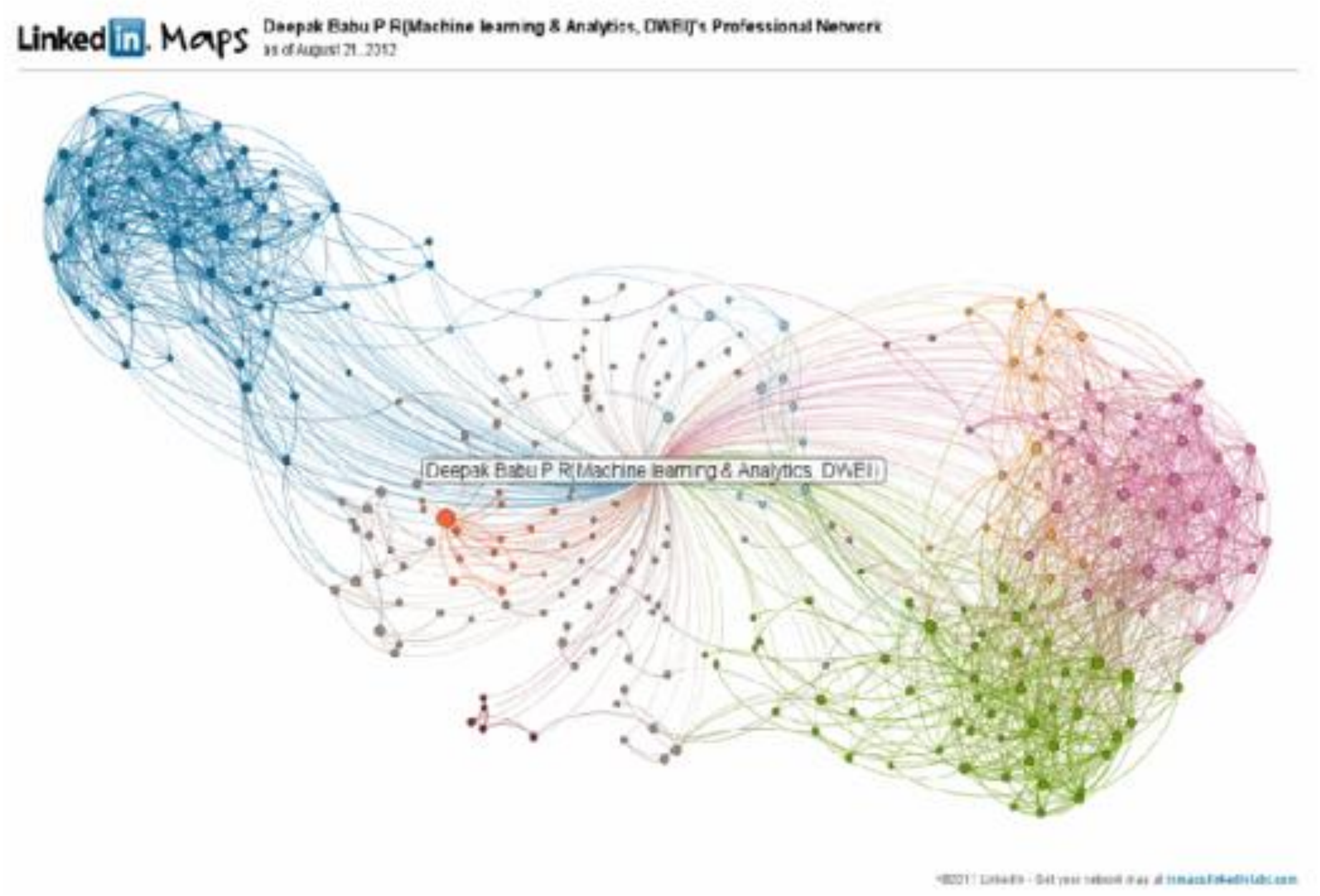
Nodes: users

Edges: interactions

Edge classification:
Predict whether there are missing links between two nodes
e.g., friend recommendation

Image credit https://prdeepakbabu.wordpress.com/2012/10/06/social-network-analytics-visualization/

Social network



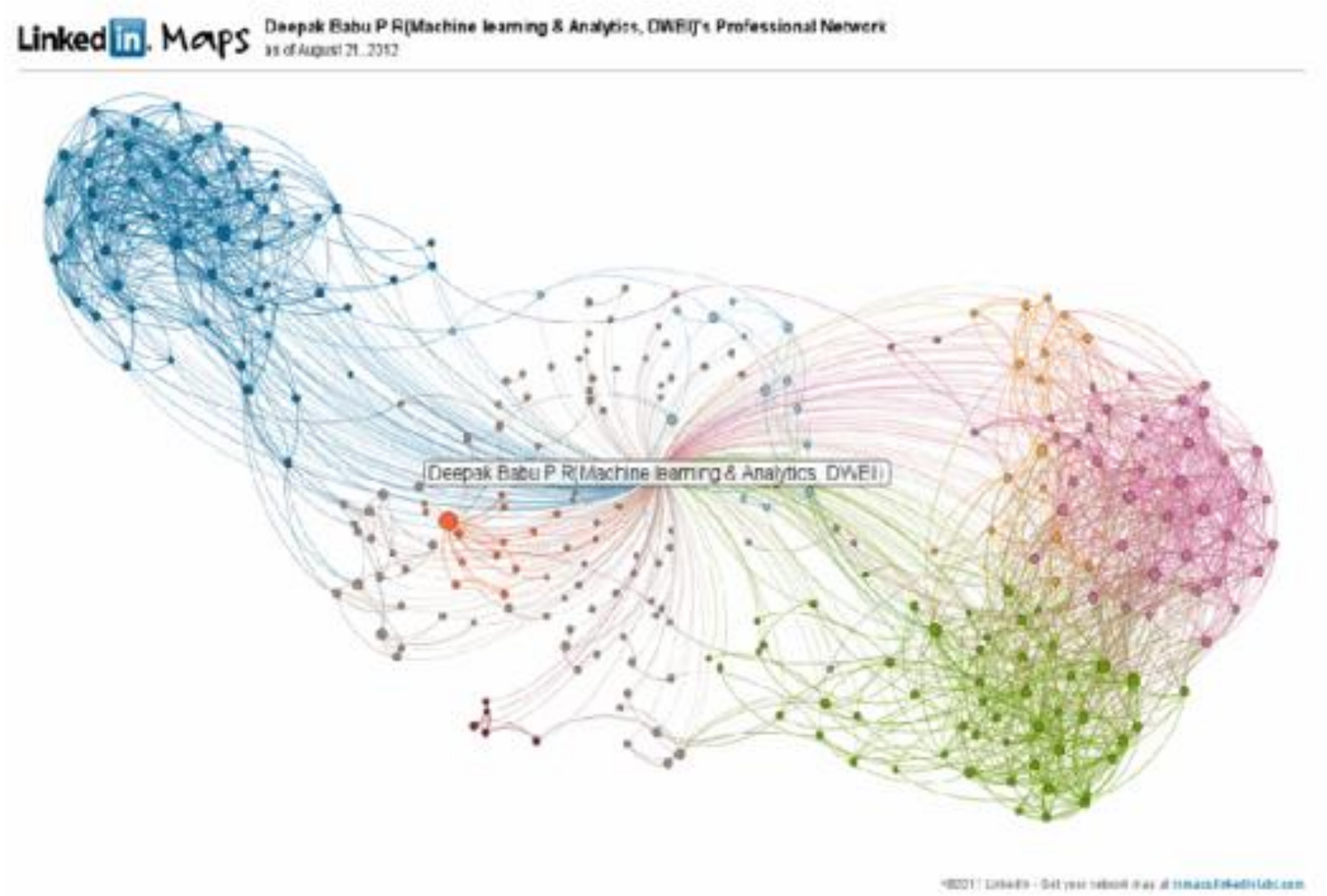
Nodes: users

Edges: interactions

Community detection:
Discover and group nodes tightly connected

Image credit https://prdeepakbabu.wordpress.com/2012/10/06/social-network-analytics-visualization/

Social network



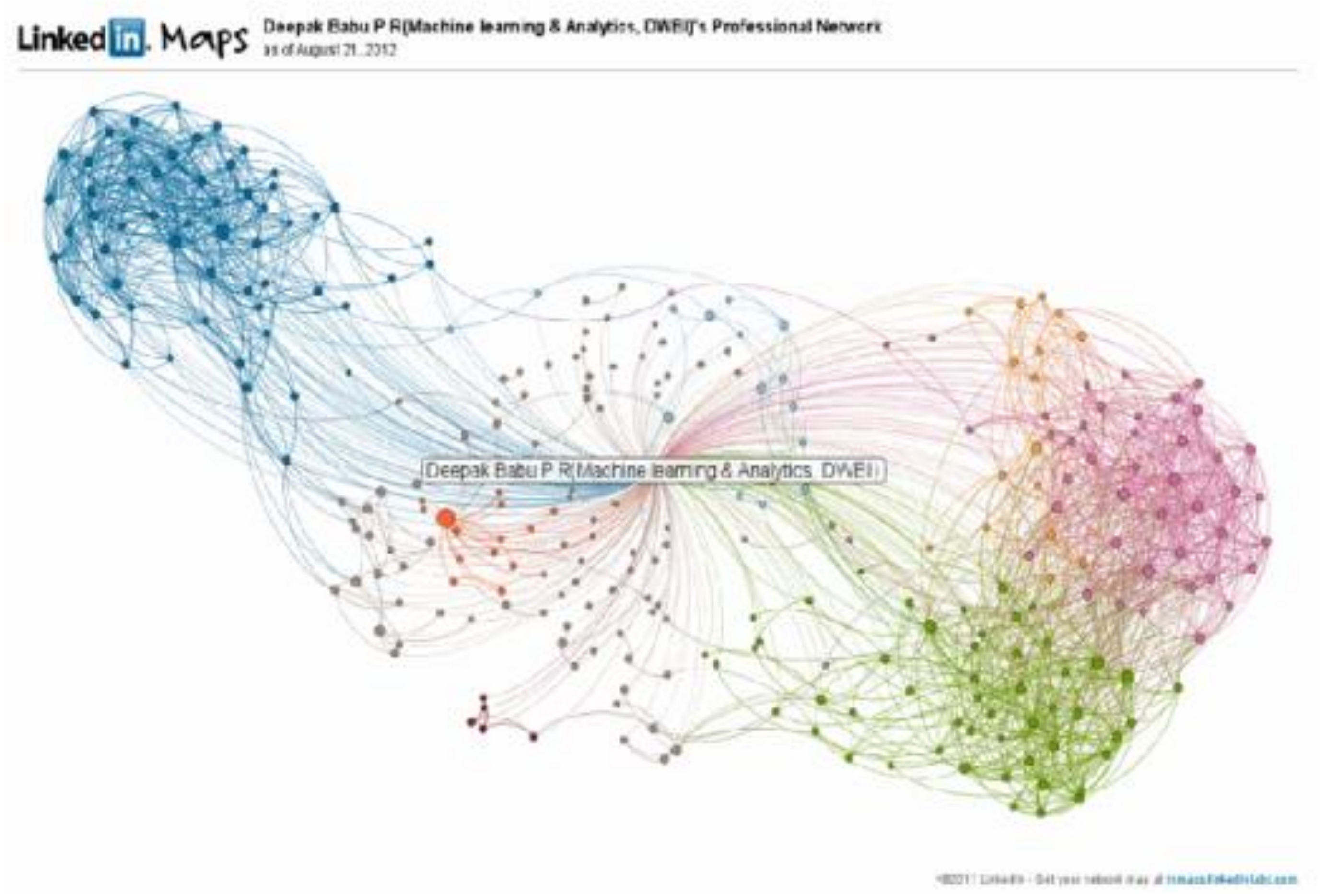
Nodes: users

Edges: interactions

Graph classification:
Categorize different graphs

Image credit https://prdeepakbabu.wordpress.com/2012/10/06/social-network-analytics-visualization/

Social network



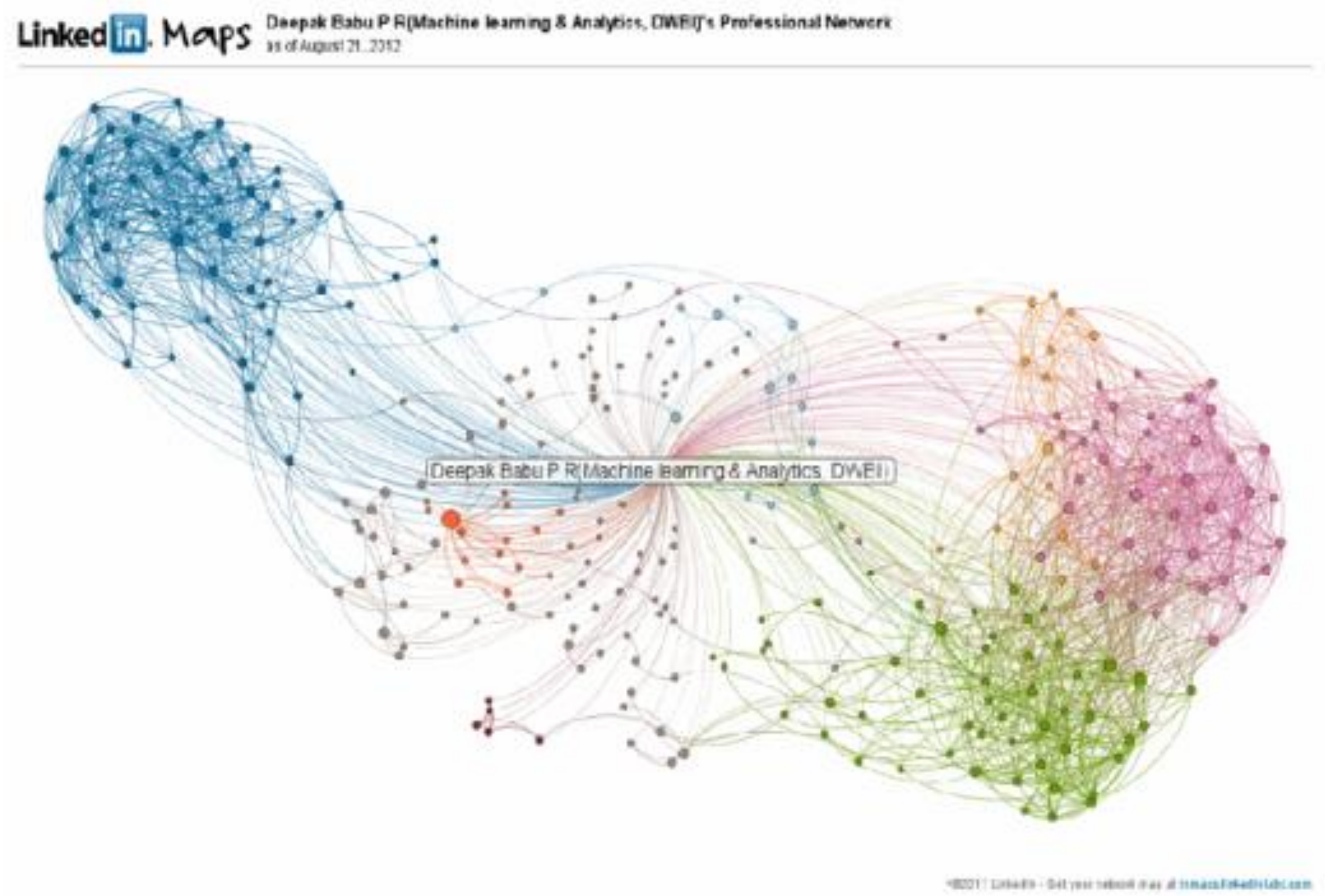
Nodes: users

Edges: interactions

Graph classification:
Categorize different graphs
e.g., Molecule property prediction

Image credit https://prdeepakbabu.wordpress.com/2012/10/06/social-network-analytics-visualization/

Social network



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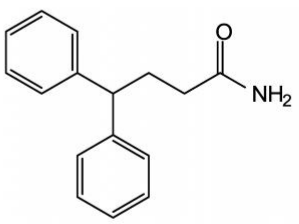
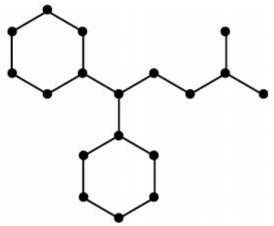
Graph classification:
Categorize different graphs
e.g., Molecule property prediction





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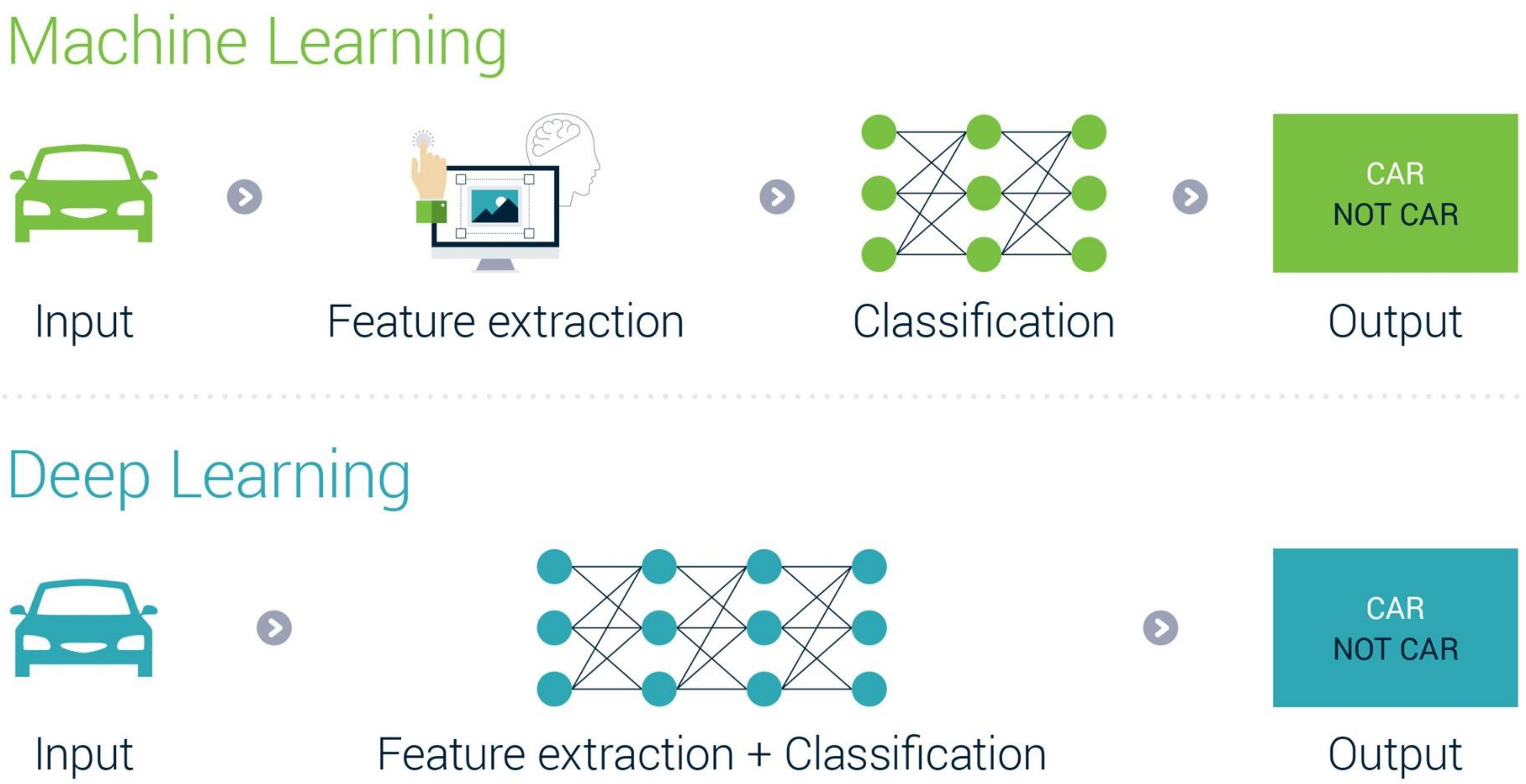
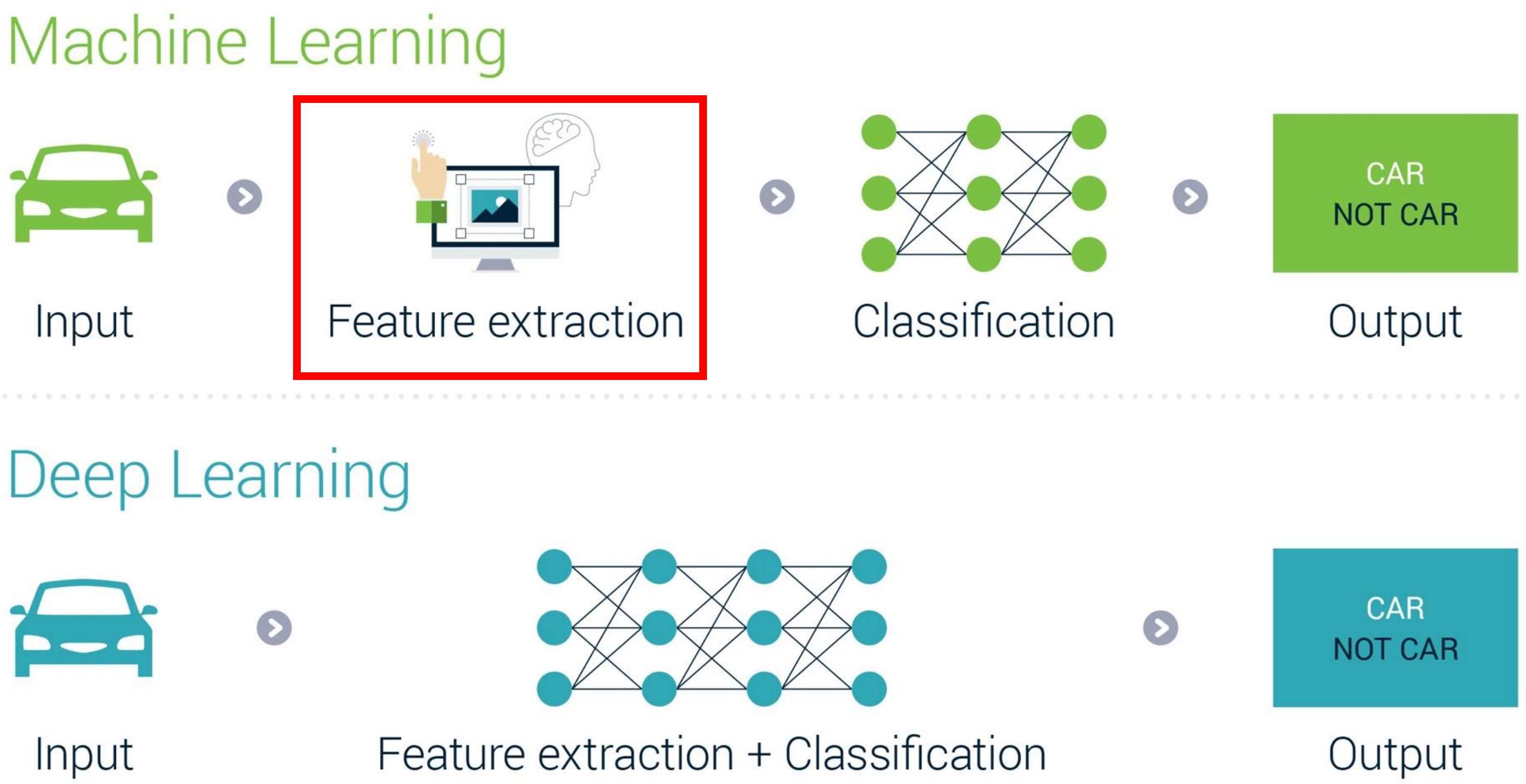
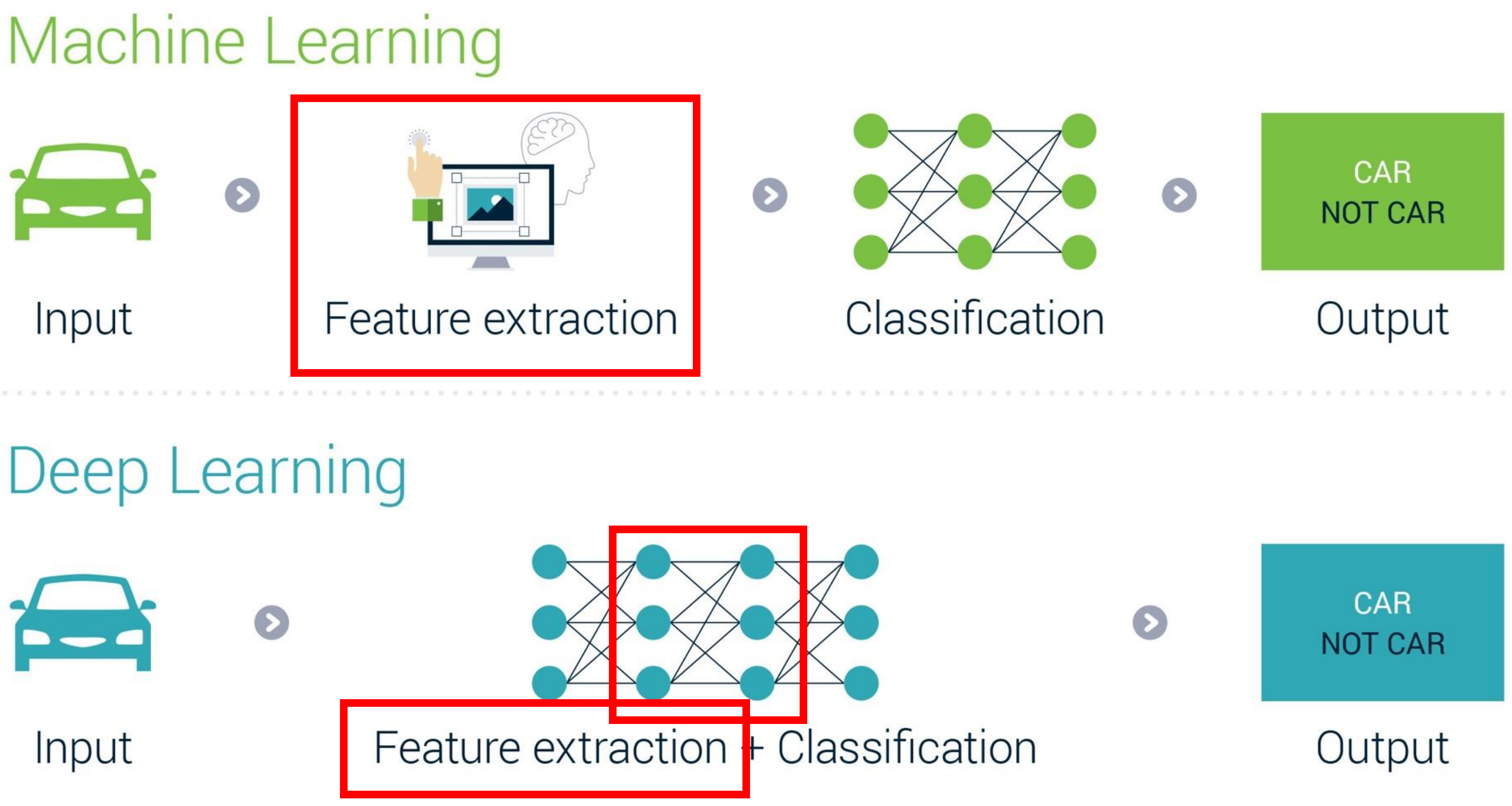
Machine learning tasks for graph data



Machine learning tasks for graph data

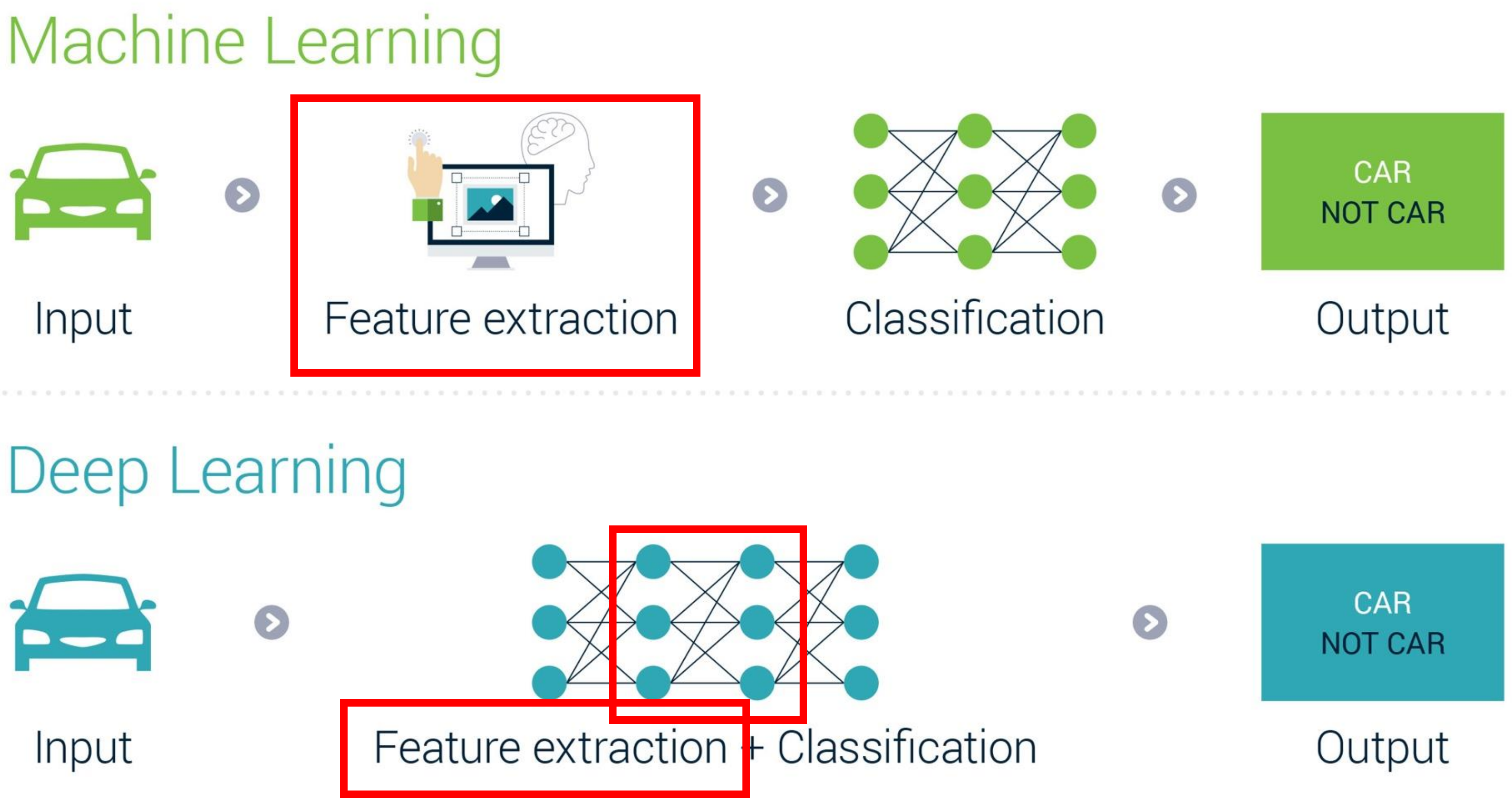


Machine learning tasks for graph data



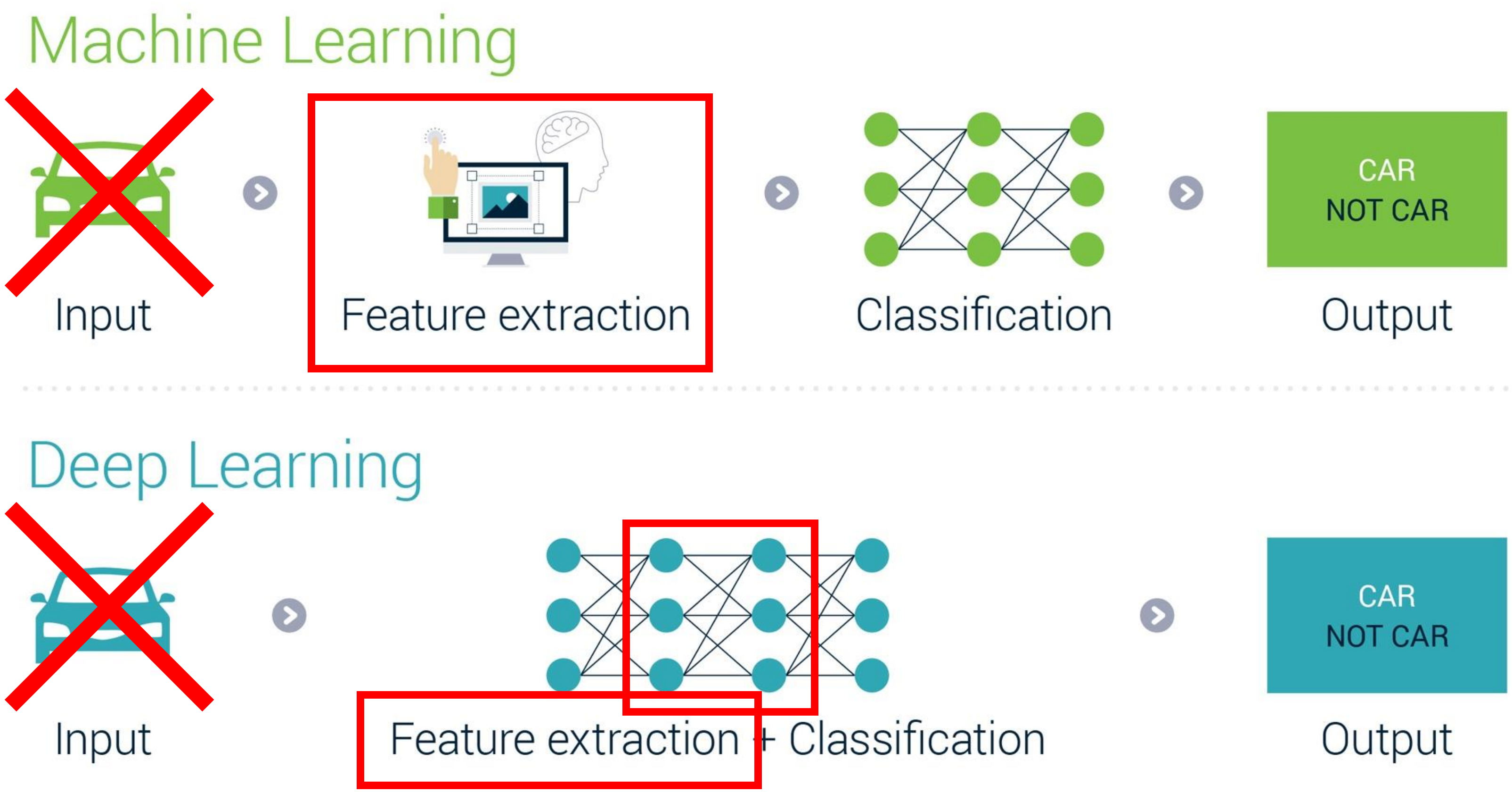
Machine learning tasks for graph data

Consider correlations



Machine learning tasks for graph data

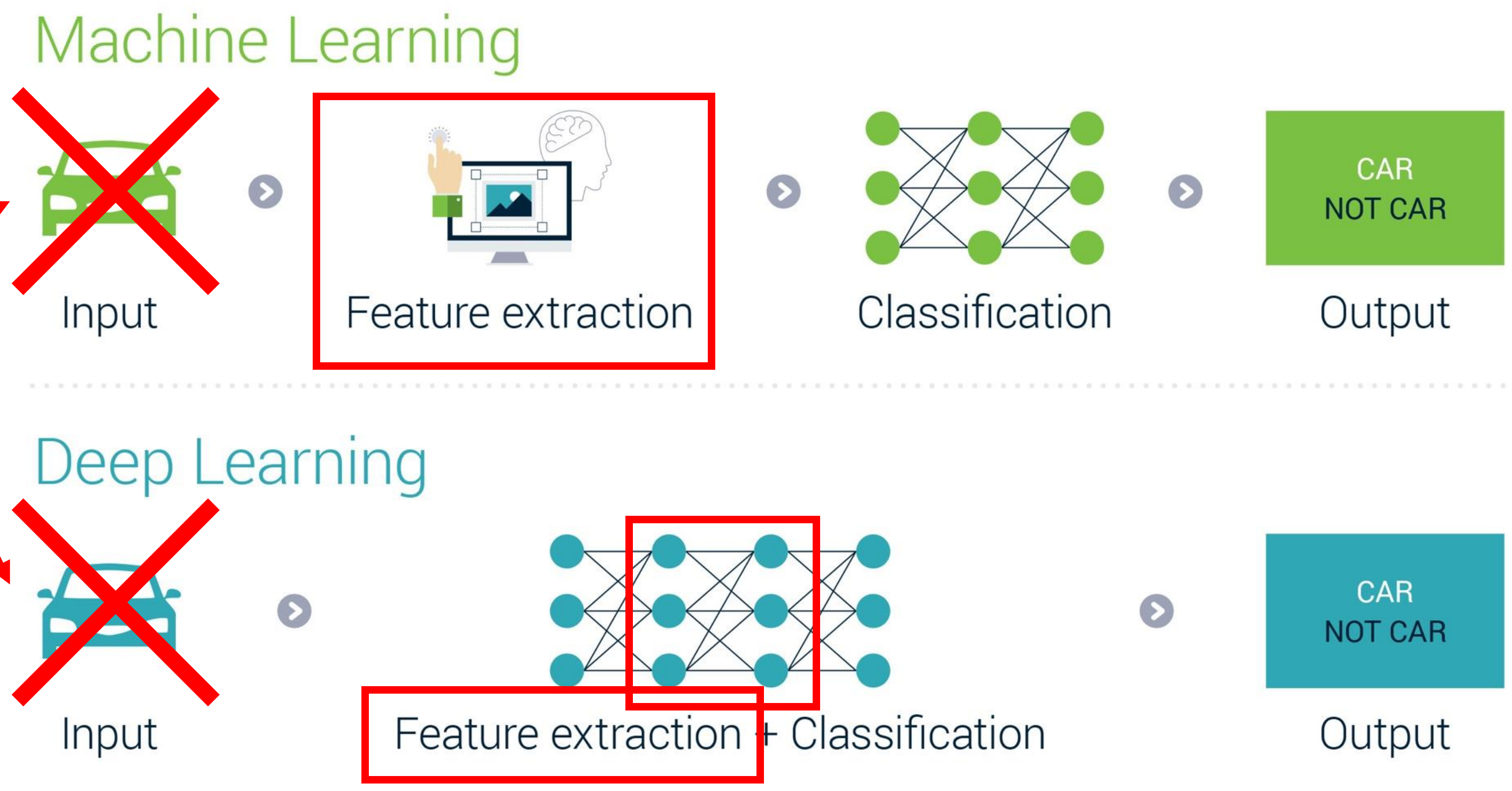
Consider correlations



Machine learning tasks for graph data

Consider correlations

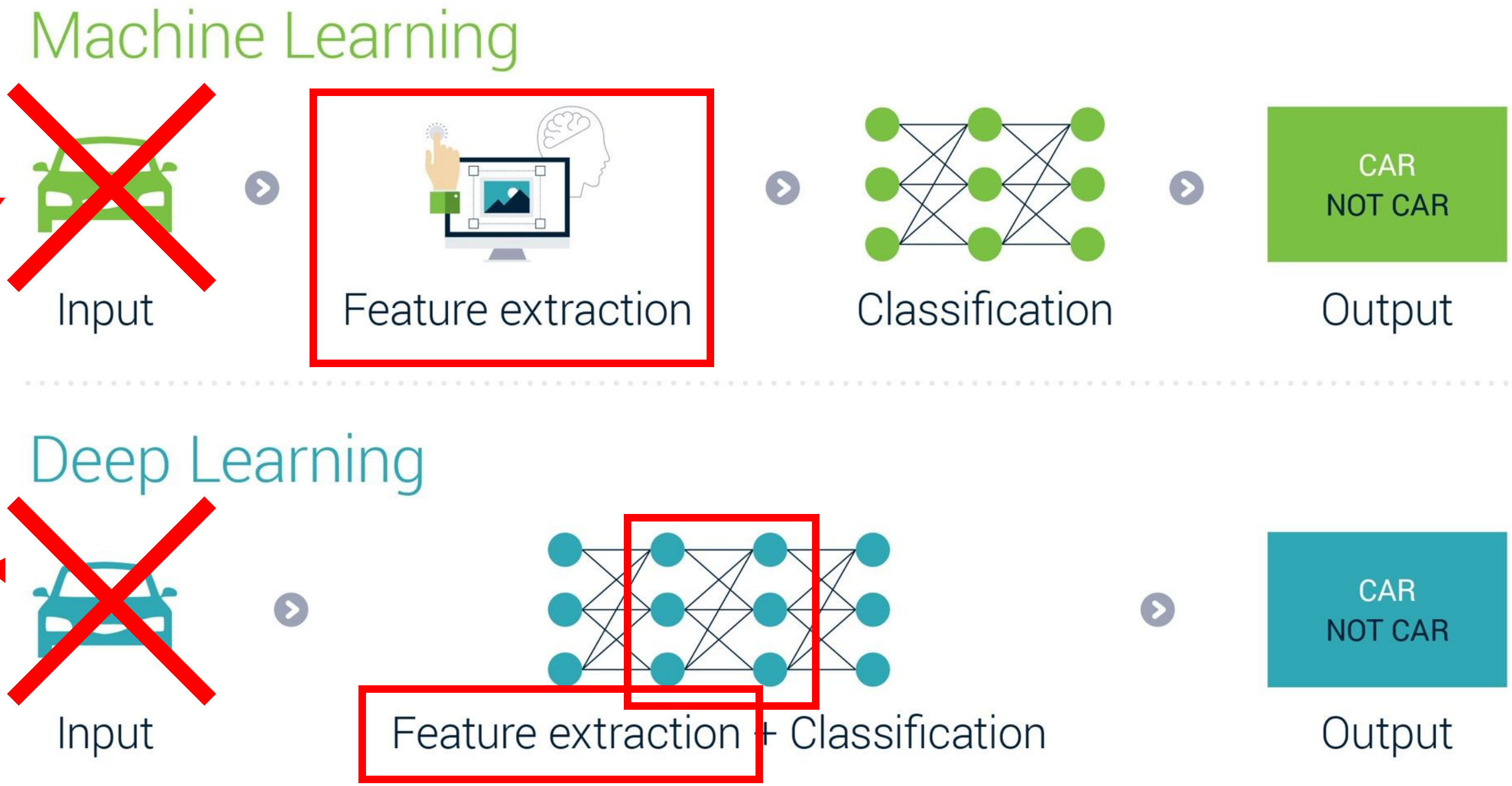
Graph data



Machine learning tasks for graph data

Consider correlations

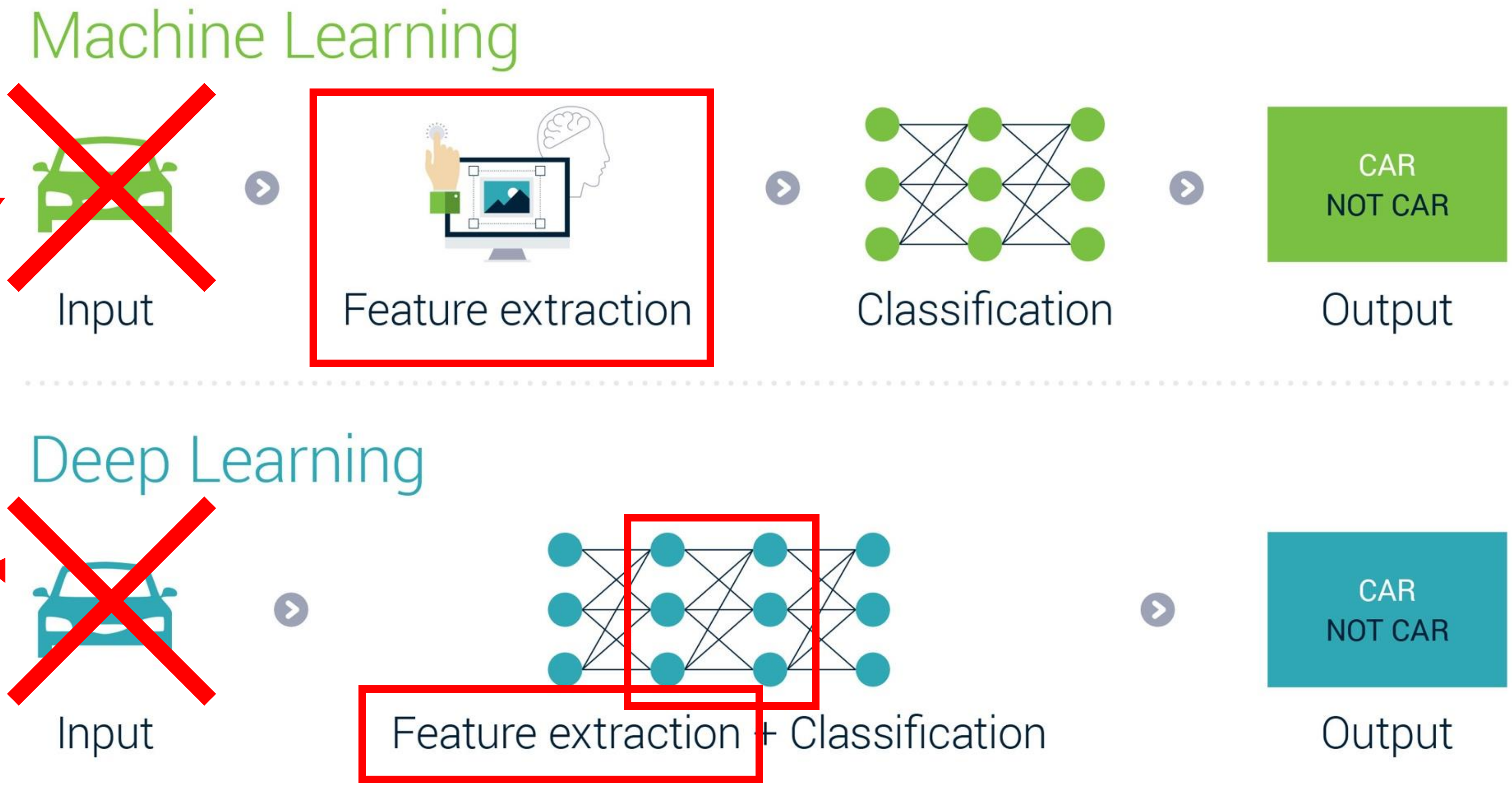
Graph data:
Nodes
Edges
Communities
Graphs



Machine learning tasks for graph data

Consider correlations

Graph data:
Nodes
Edges
Communities
Graphs

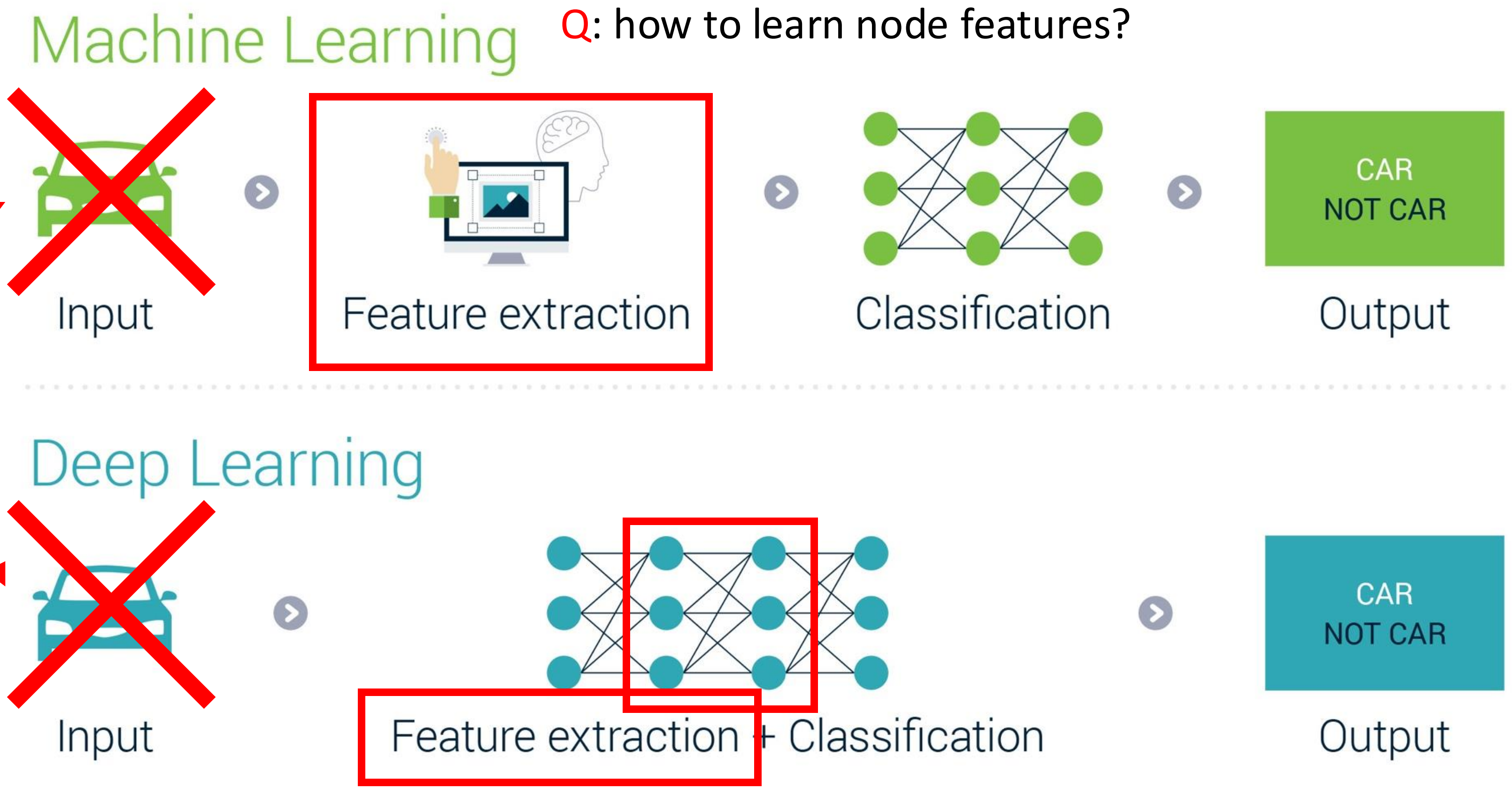


Machine learning tasks for graph data

Q: how to learn node features?

Consider correlations

Graph data:
Nodes
Edges
Communities
Graphs

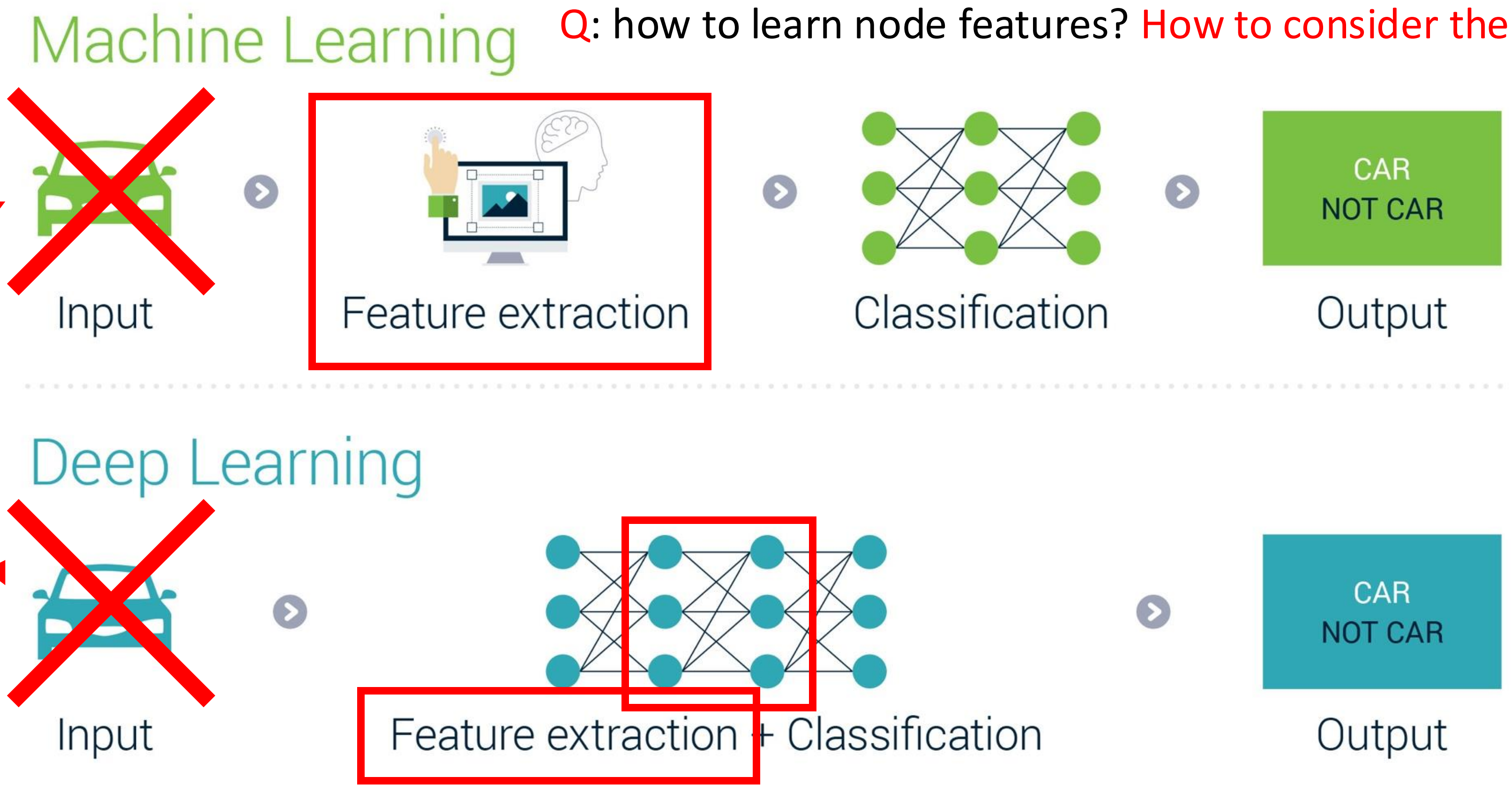


Machine learning tasks for graph data

Q: how to learn node features? How to consider the correlation in graph?

Consider correlations

Graph data:
Nodes
Edges
Communities
Graphs

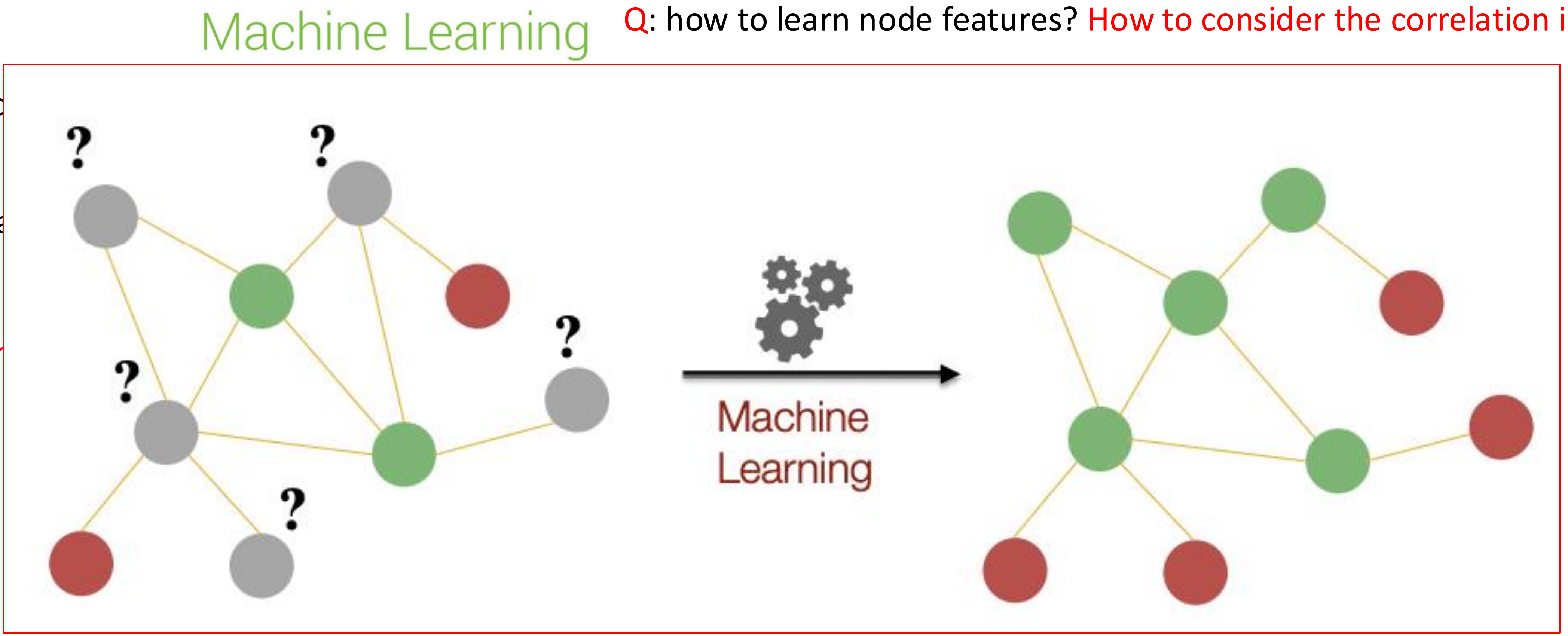


Machine learning tasks for graph data

Q: how to learn node features? How to consider the correlation in graph?

Consider d

Graph da
Nodes
Edges
Commun
Graphs

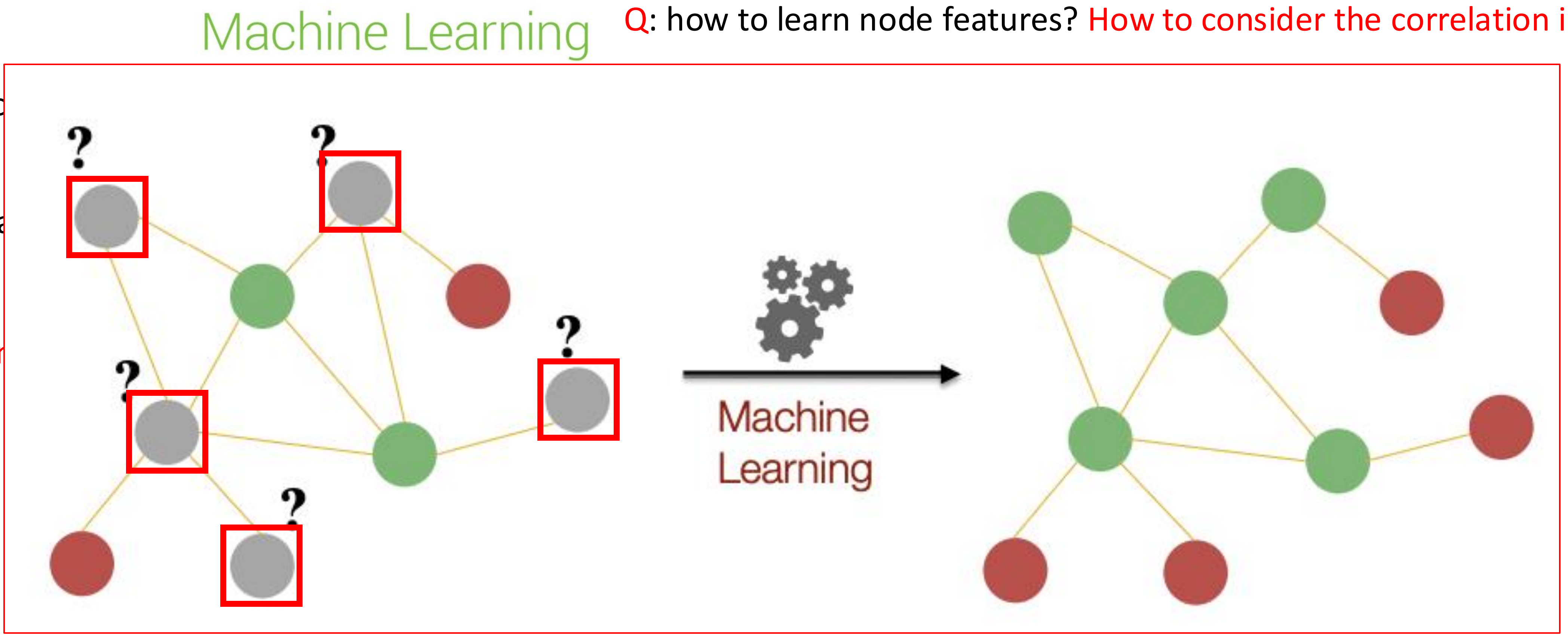


Machine learning tasks for graph data

Q: how to learn node features? How to consider the correlation in graph?

Consider d

Graph da
Nodes
Edges
Commun
Graphs

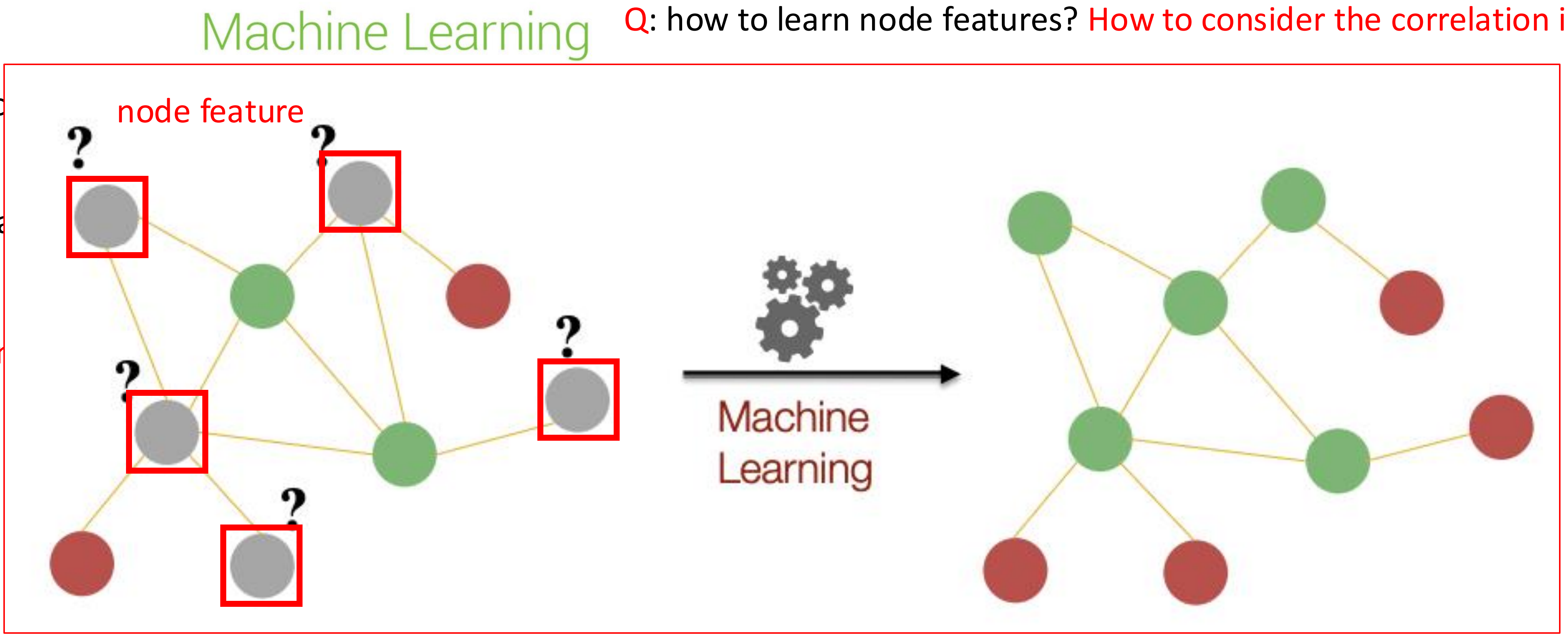


Machine learning tasks for graph data

Q: how to learn node features? How to consider the correlation in graph?

Consider d

Graph da
Nodes
Edges
Commun
Graphs

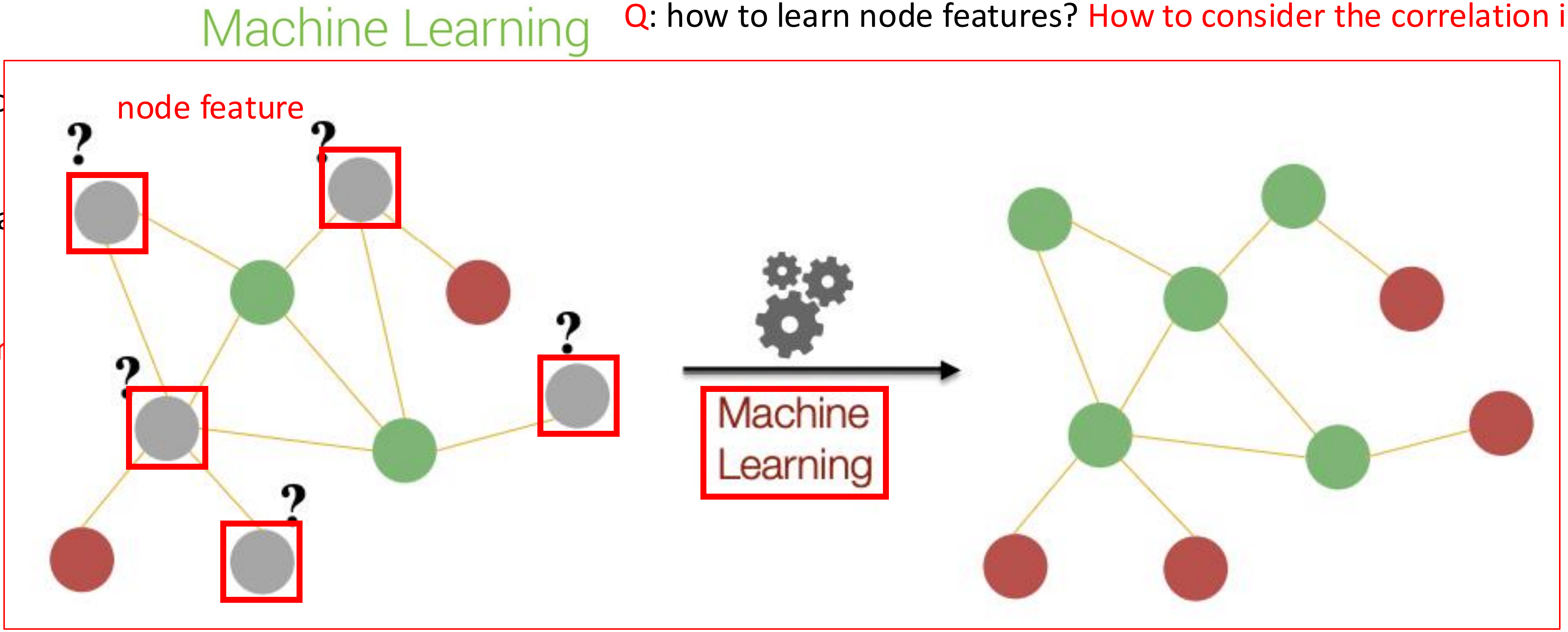


Machine learning tasks for graph data

Q: how to learn node features? How to consider the correlation in graph?

Consider d

Graph da
Nodes
Edges
Commun
Graphs

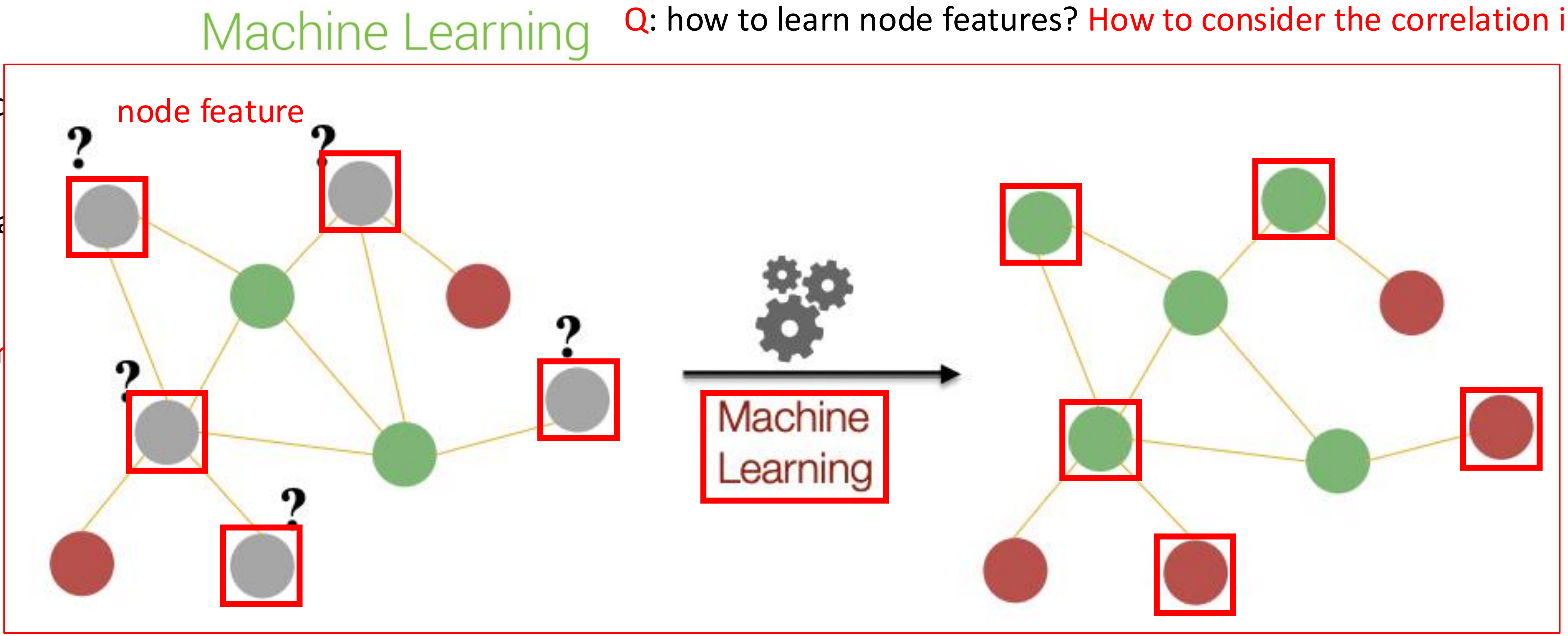


Machine learning tasks for graph data

Q: how to learn node features? How to consider the correlation in graph?

Consider d

Graph da
Nodes
Edges
Commun
Graphs



Node degree

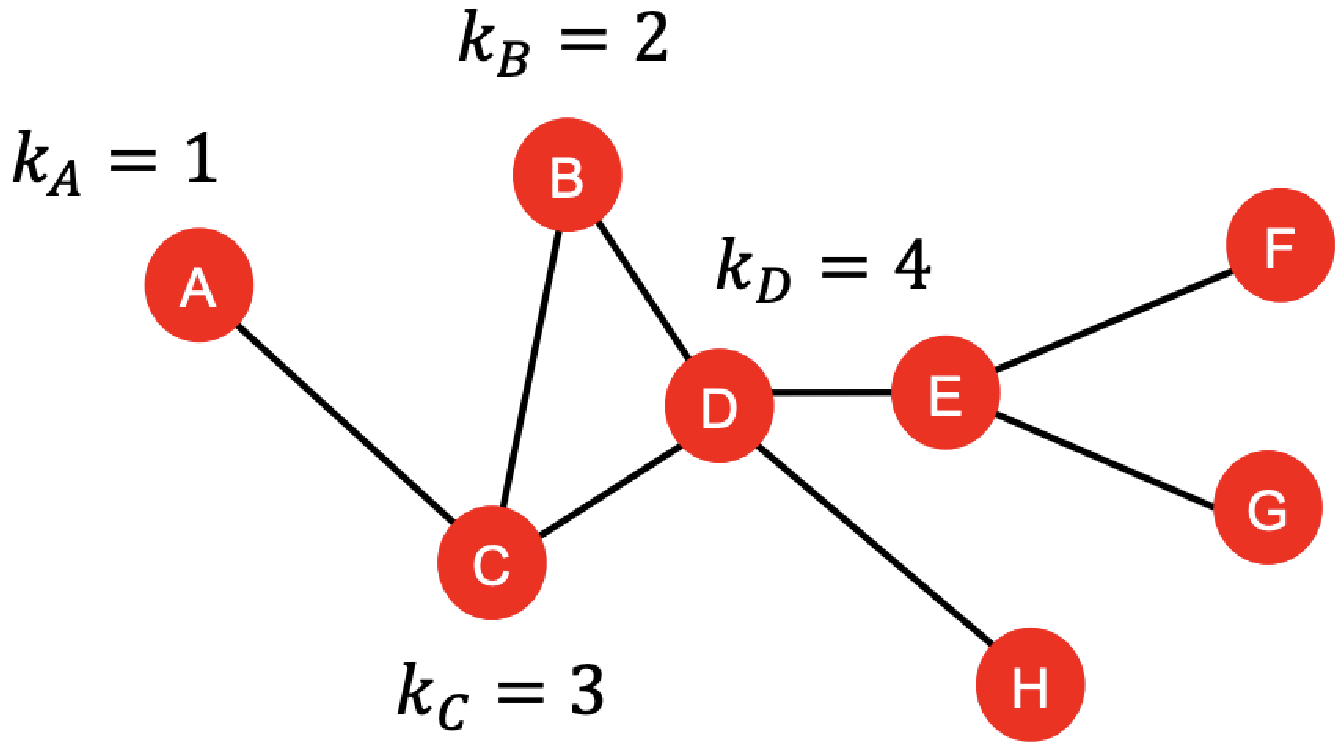


Image credit http://web.stanford.edu/class/cs224w/slides/03-nodeemb.pdf

Node degree

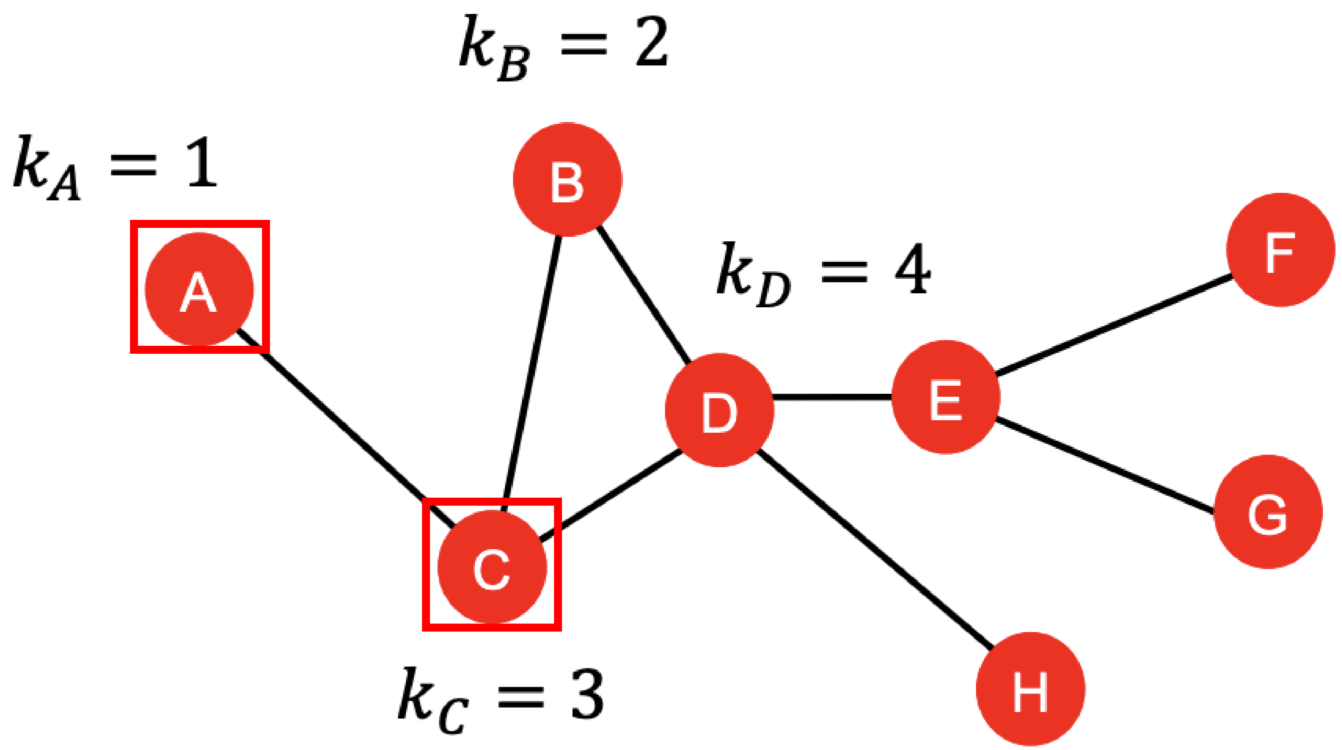


Image credit http://web.stanford.edu/class/cs224w/slides/03-nodeemb.pdf

Node degree

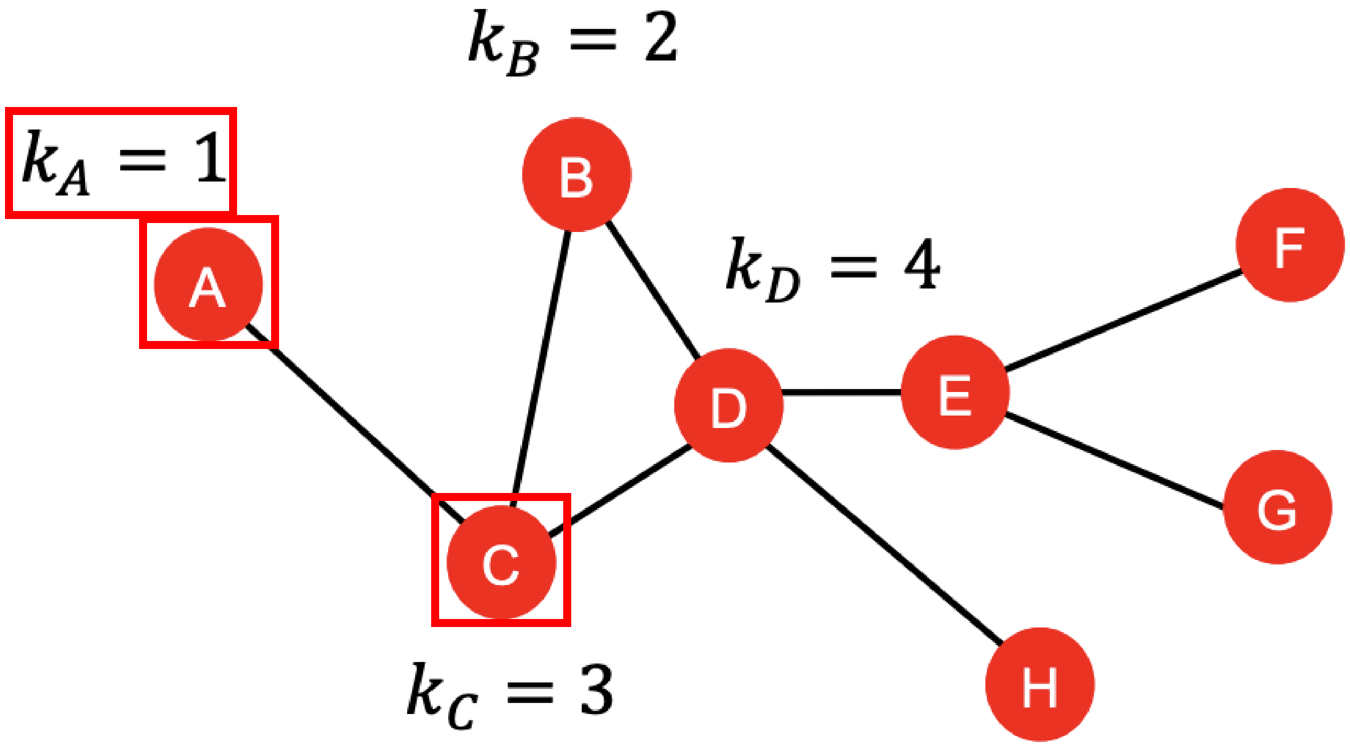


Image credit http://web.stanford.edu/class/cs224w/slides/03-nodeemb.pdf

Node degree

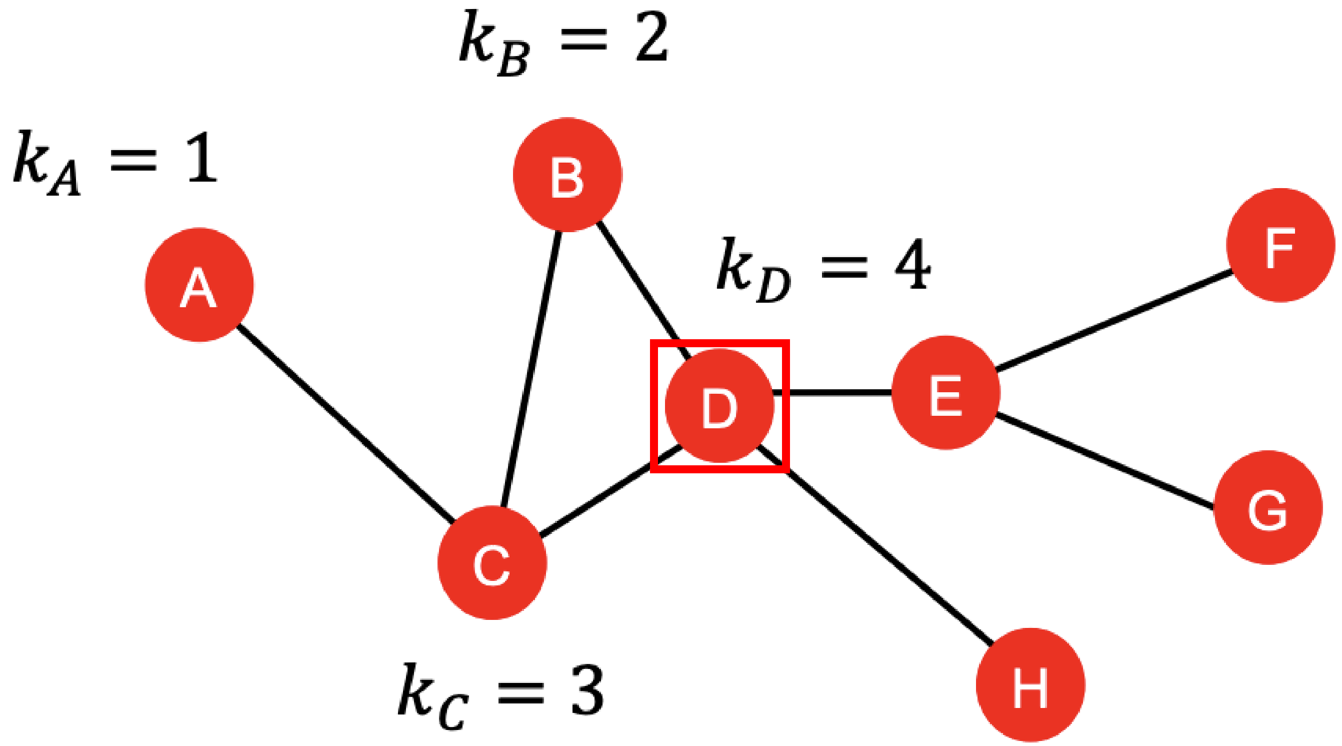


Image credit http://web.stanford.edu/class/cs224w/slides/03-nodeemb.pdf

Node degree

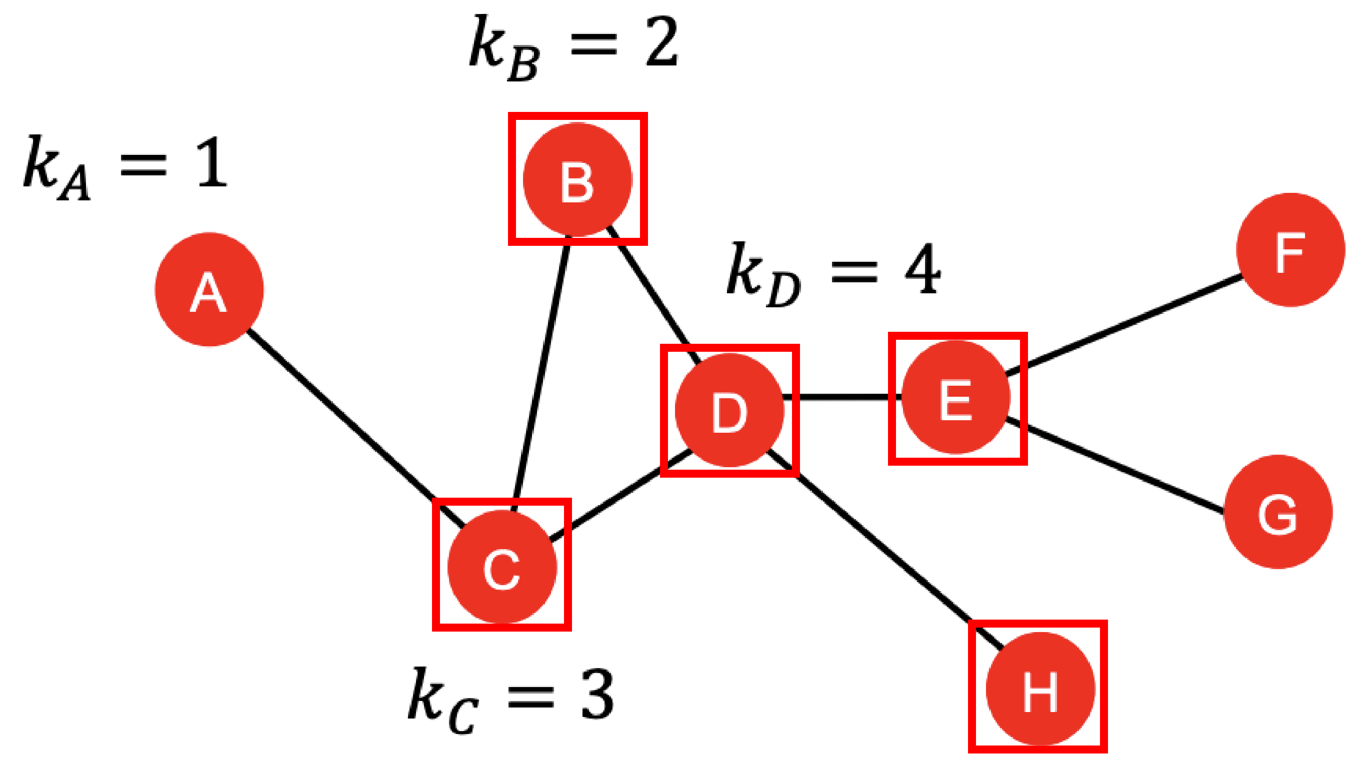


Image credit http://web.stanford.edu/class/cs224w/slides/03-nodeemb.pdf

Node degree

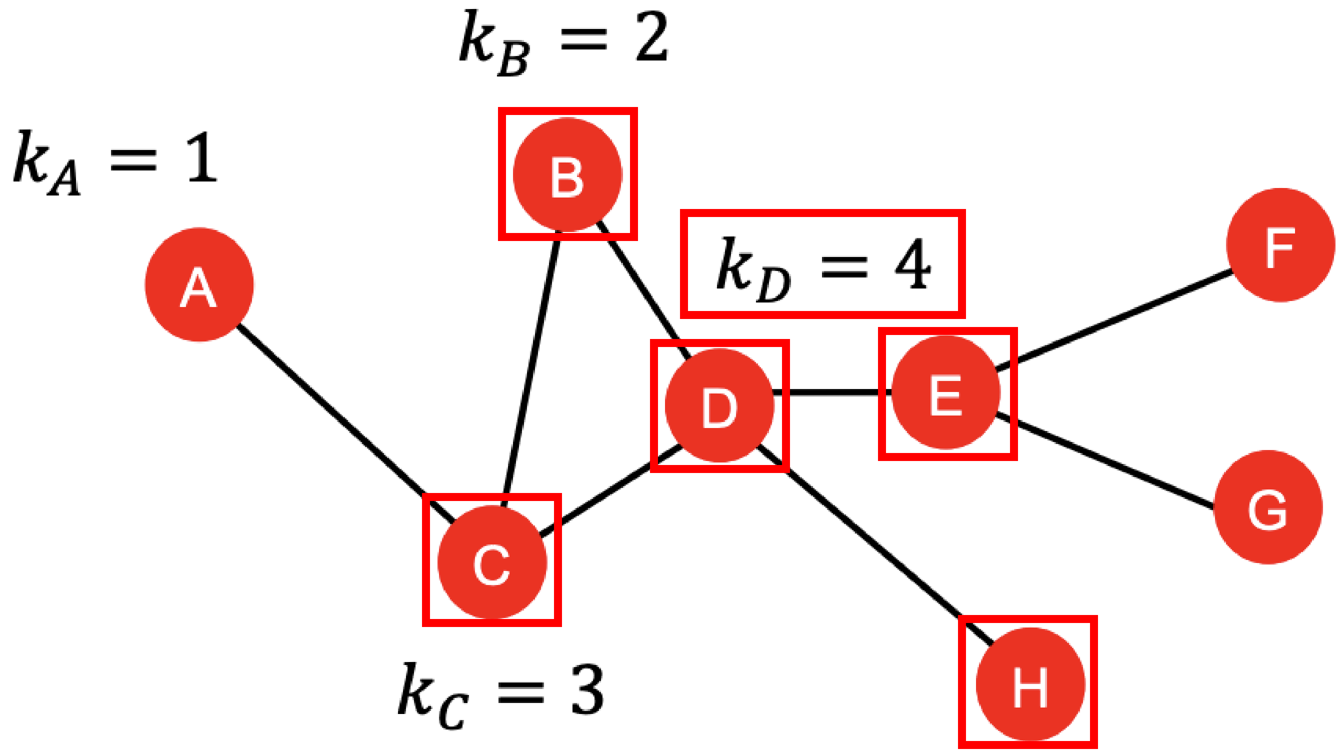
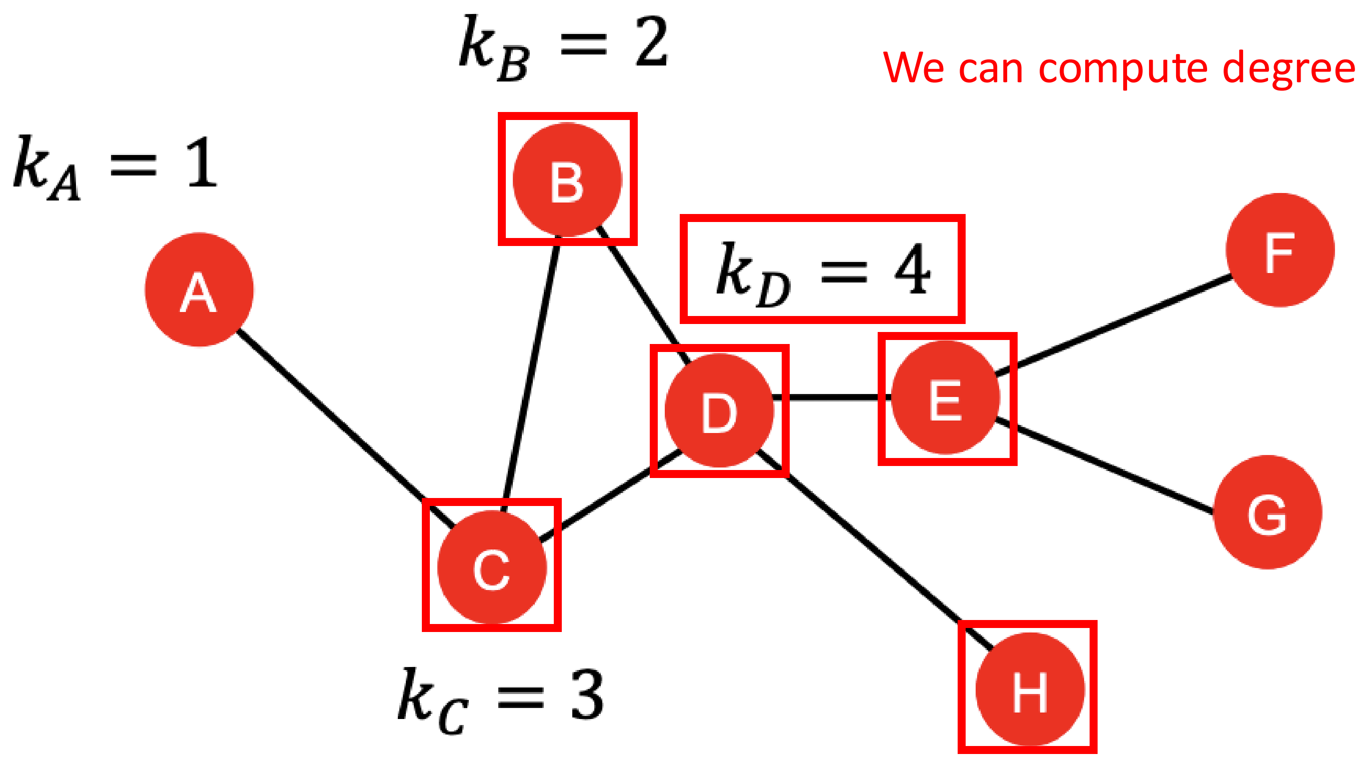


Image credit http://web.stanford.edu/class/cs224w/slides/03-nodeemb.pdf

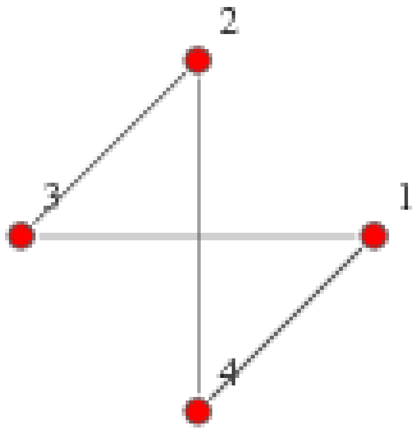
Node degree

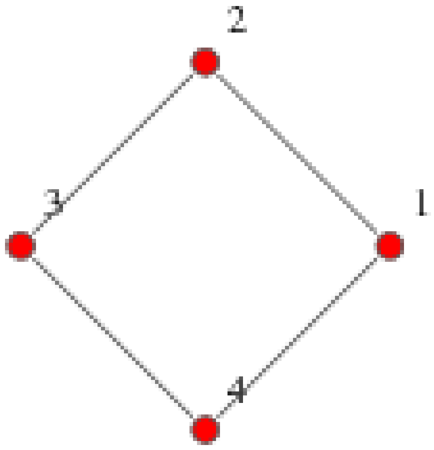


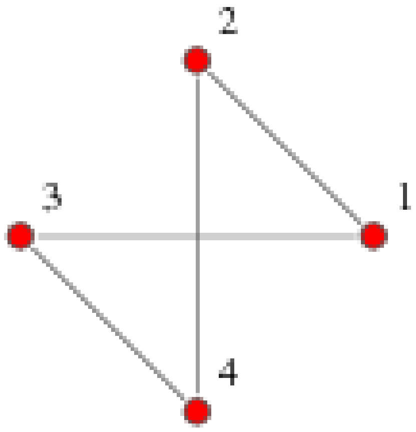
We can compute degree for each node

Image credit http://web.stanford.edu/class/cs224w/slides/03-nodeemb.pdf

Adjacency matrix

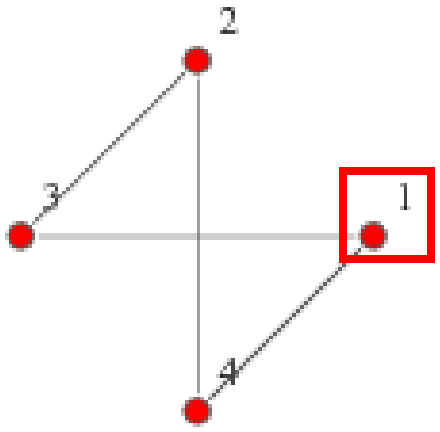


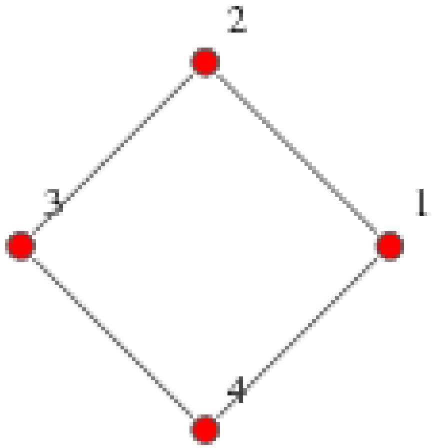


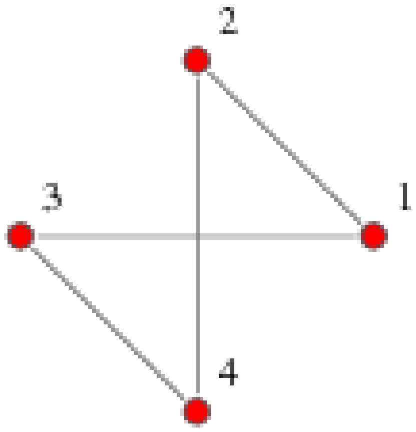


$$\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

Adjacency matrix

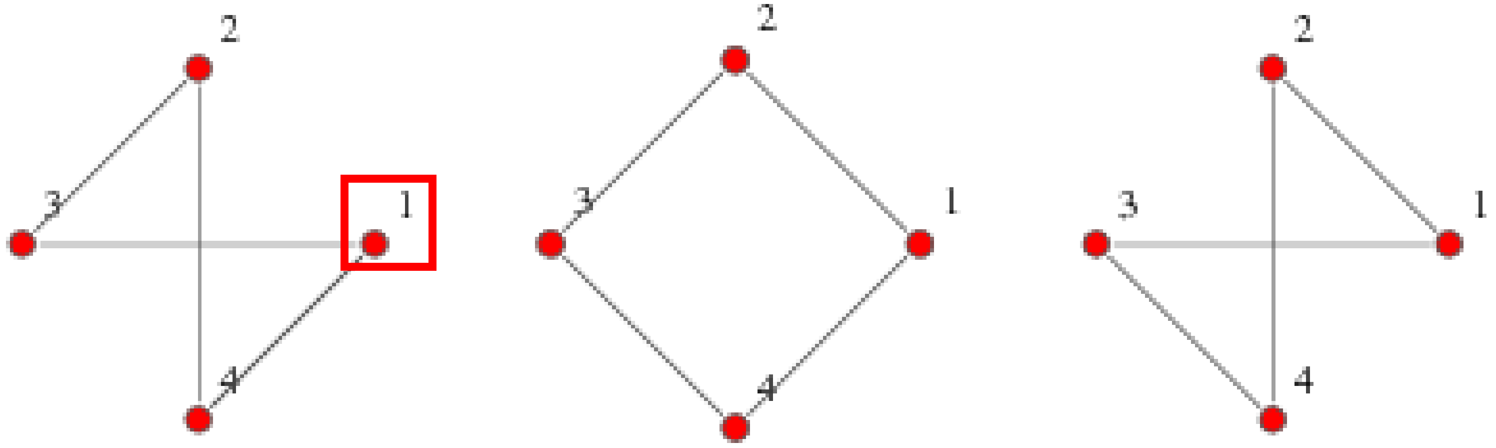






$$\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

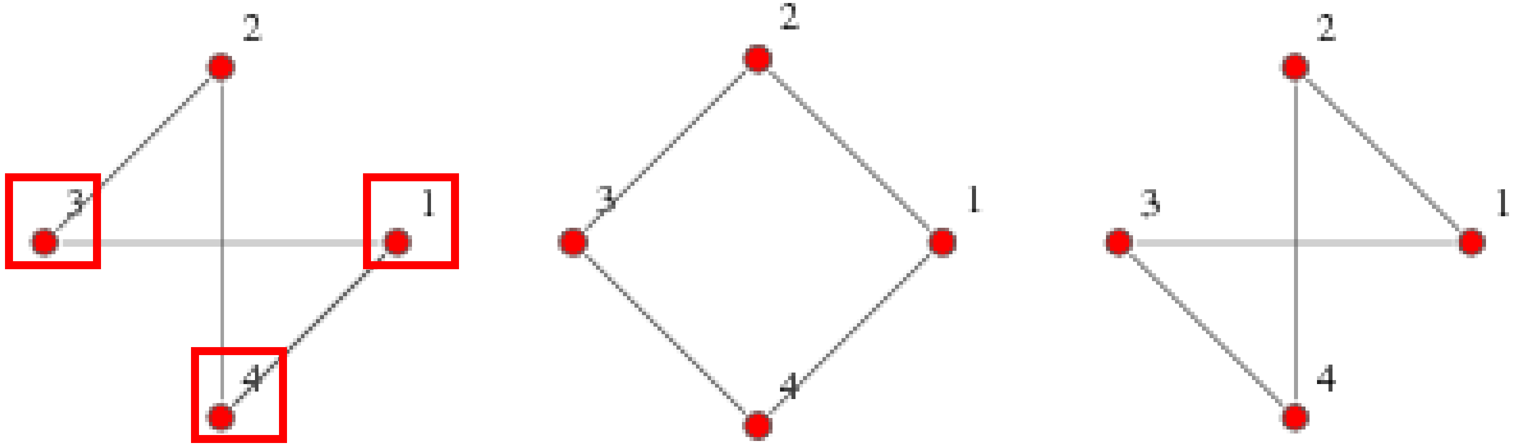
Adjacency matrix



Q: which nodes are connected to node #1?

$$
\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix}
\qquad
\begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}
\qquad
\begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}
$$

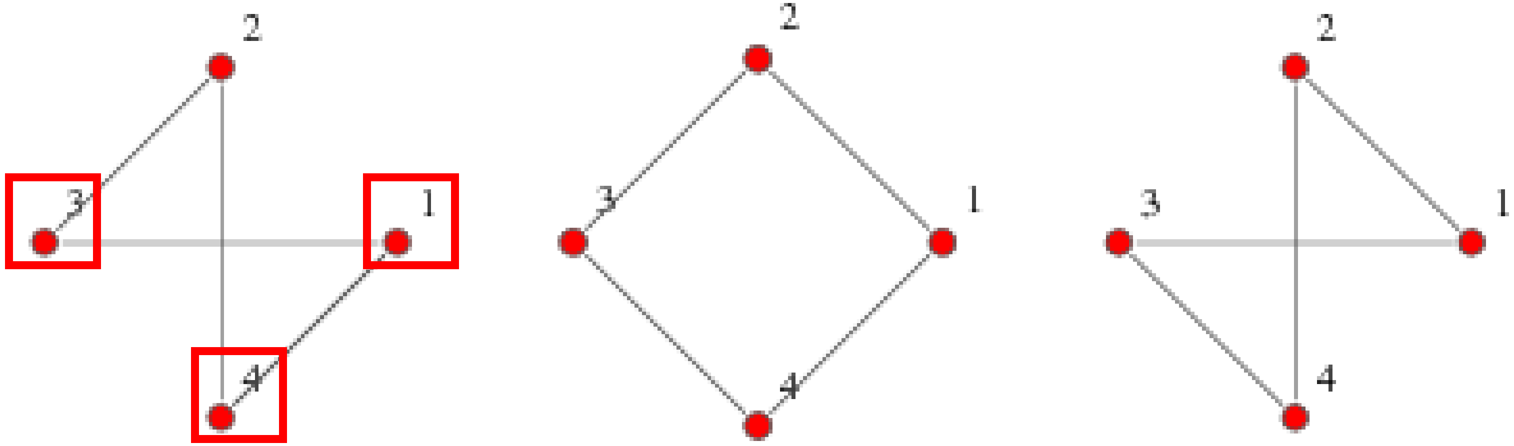
Adjacency matrix



Q: which nodes are connected to node #1?

$$\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

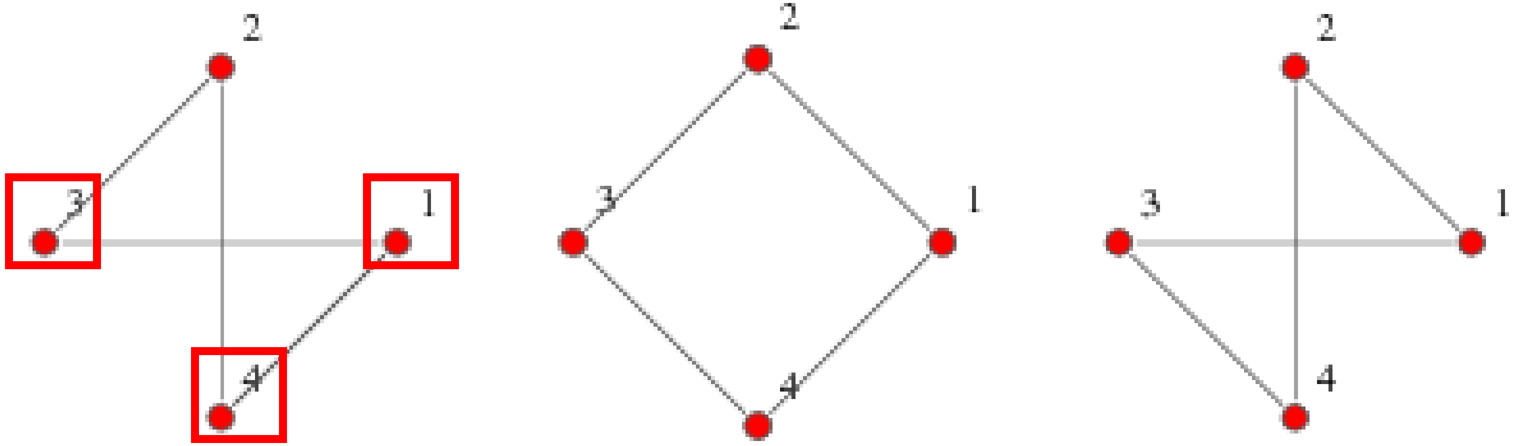
Adjacency matrix



Q: which nodes are connected to node #1?

$$\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

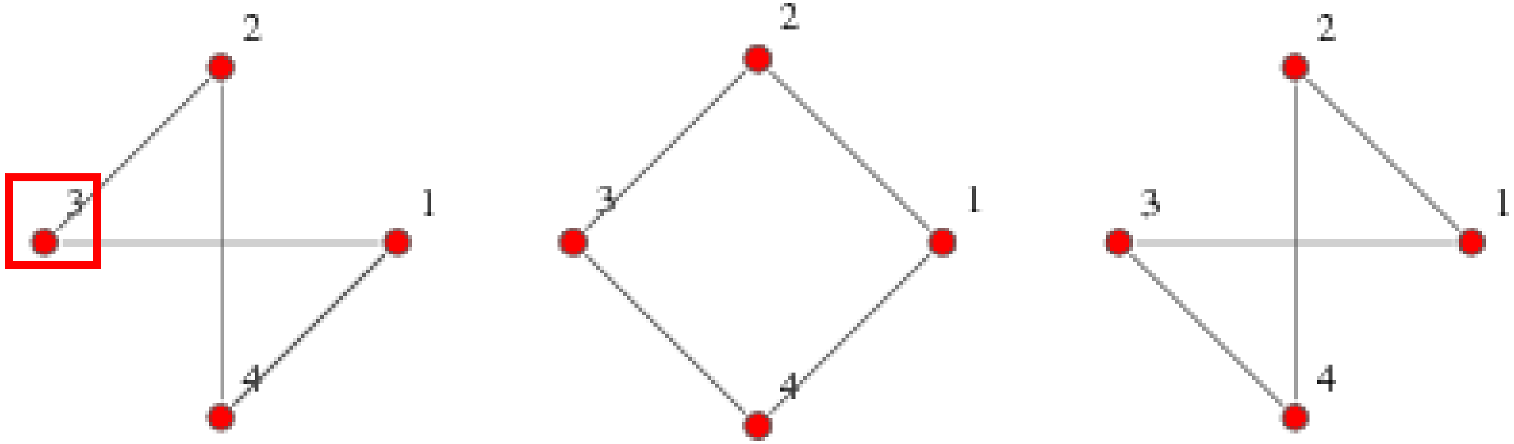
Adjacency matrix



Q: which nodes are connected to node #1?

$$\begin{pmatrix} 0 & \boxed{0} & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

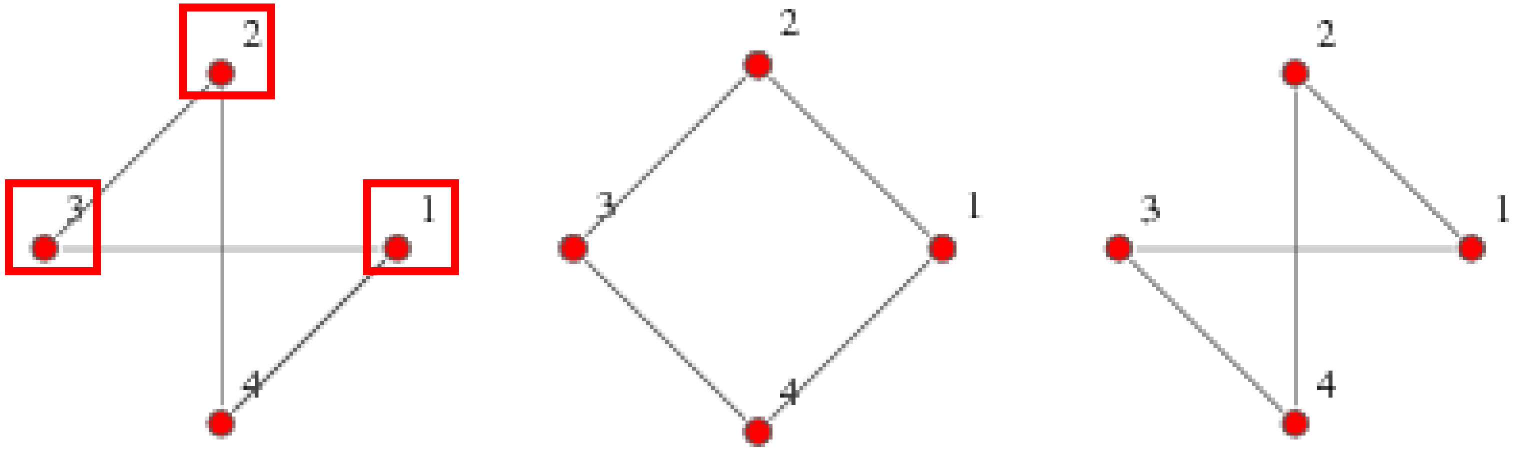
Adjacency matrix



Q: which nodes are connected to node #3?

$$\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

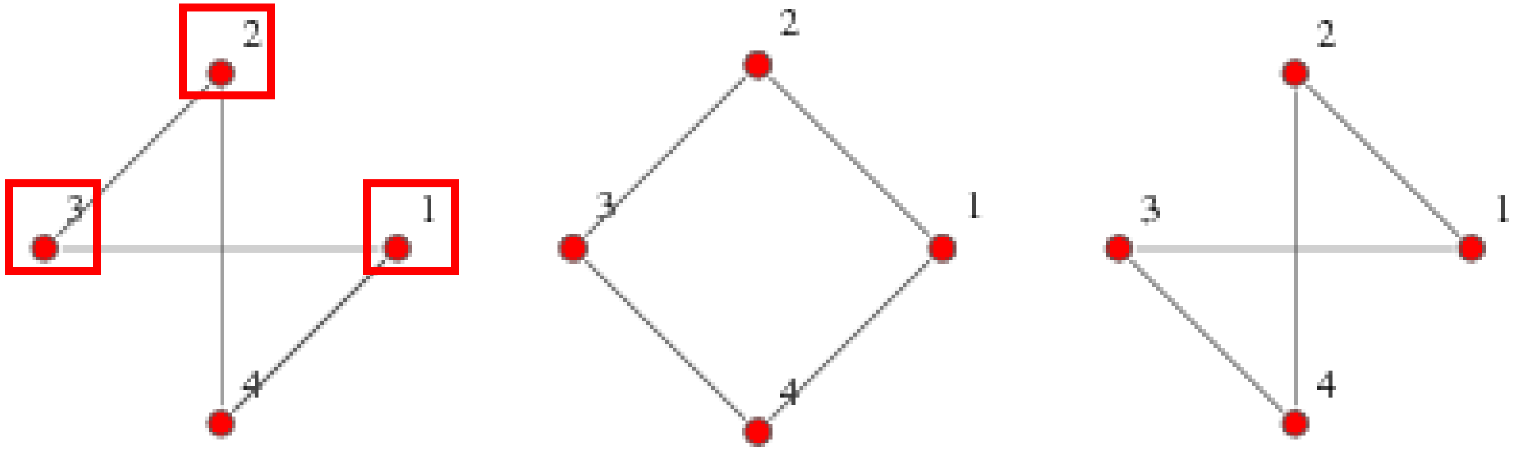
Adjacency matrix



Q: which nodes are connected to node #3?

$$\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

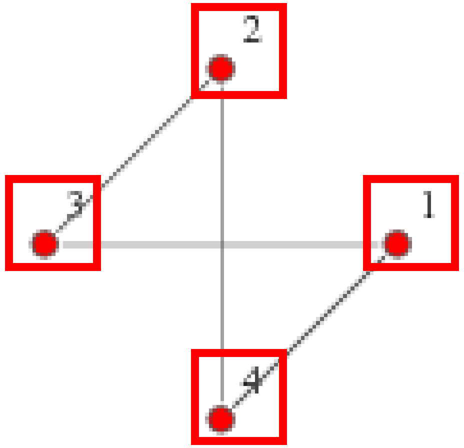
Adjacency matrix

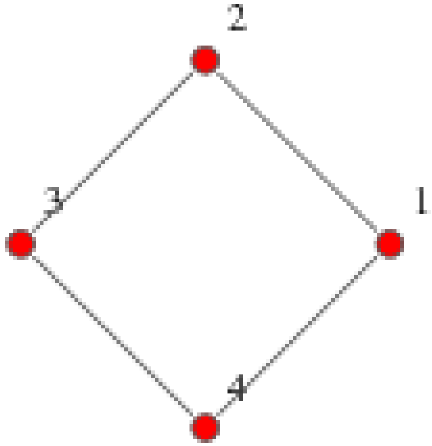


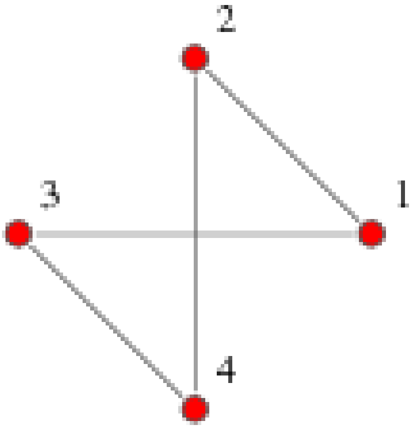
Q: which nodes are connected to node #3?

$$
\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix}
\quad
\begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}
\quad
\begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}
$$

Adjacency matrix

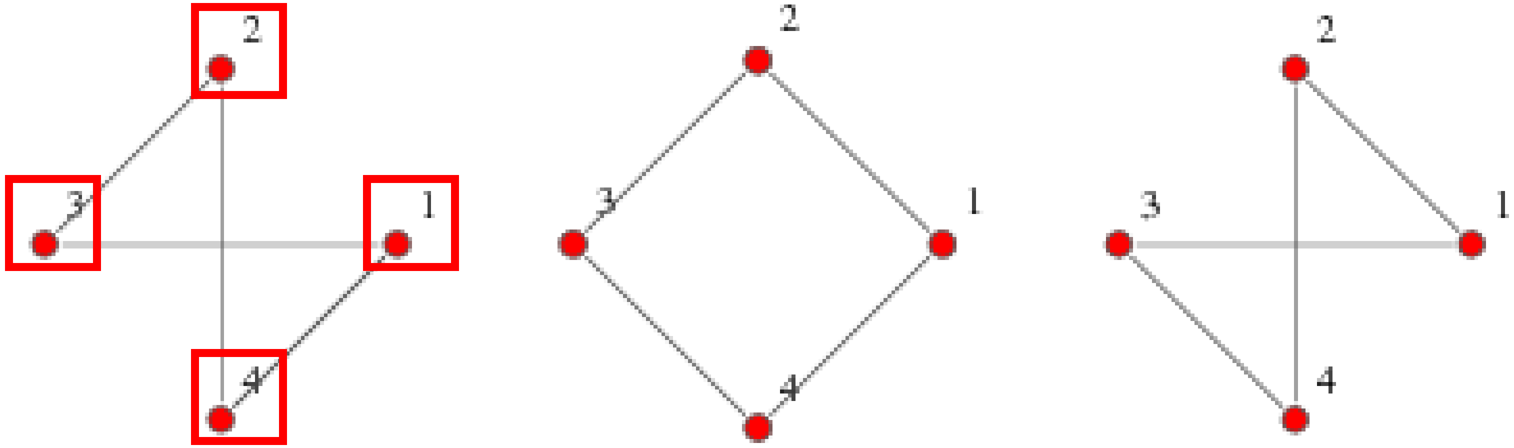






$$\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

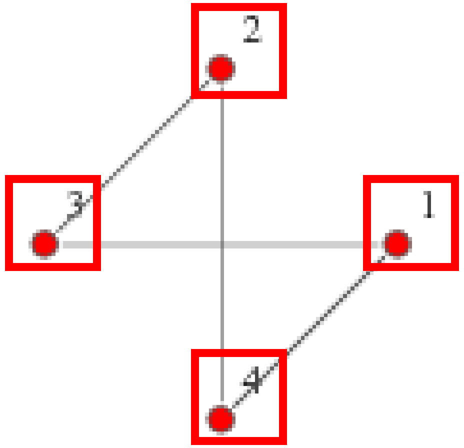
Adjacency matrix

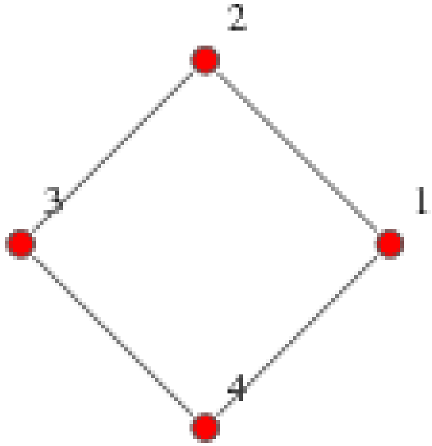


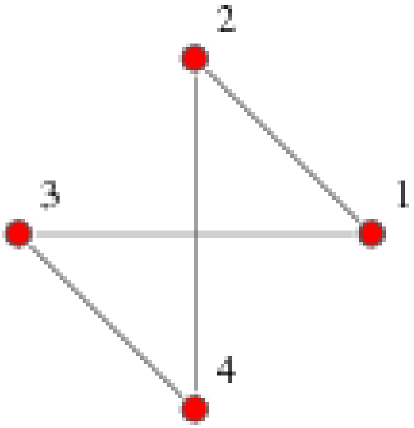
$$\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

Q: What do entries on diagonal stand for?

Adjacency matrix

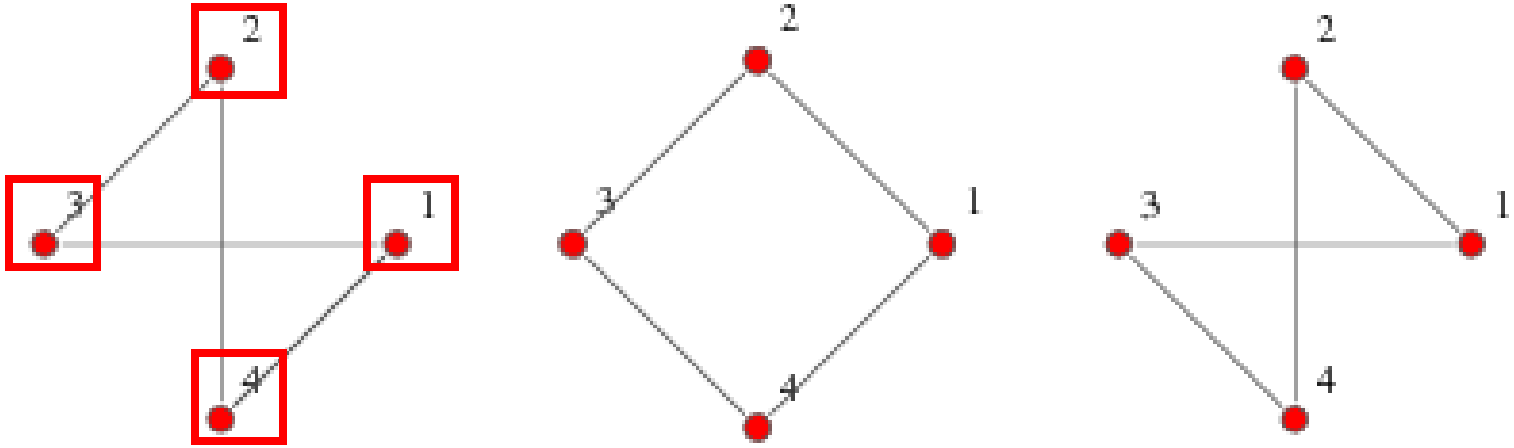






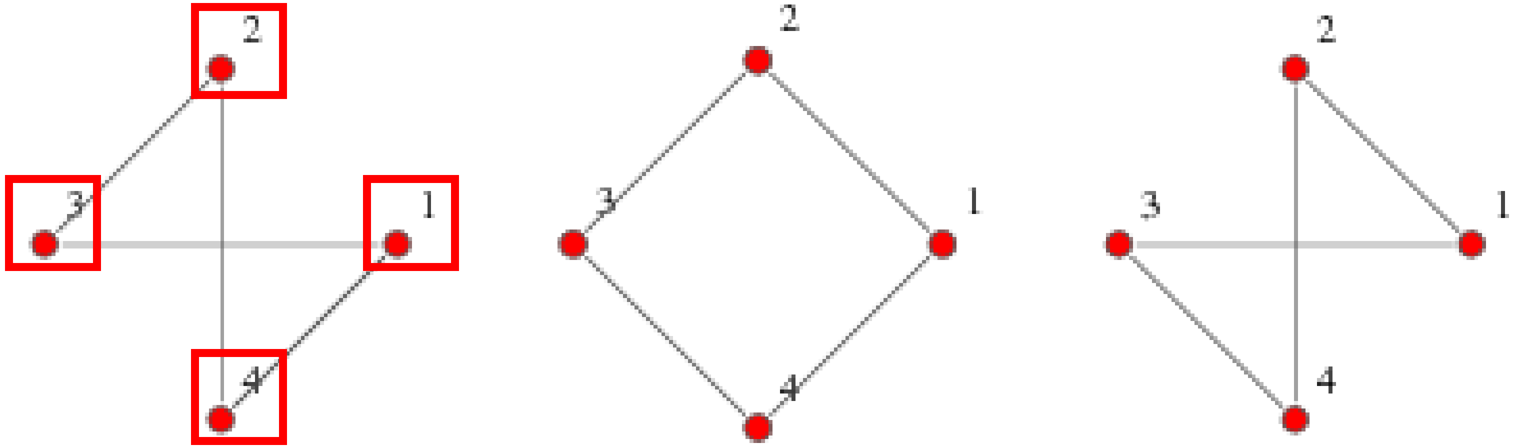
$$\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

Adjacency matrix



$$\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

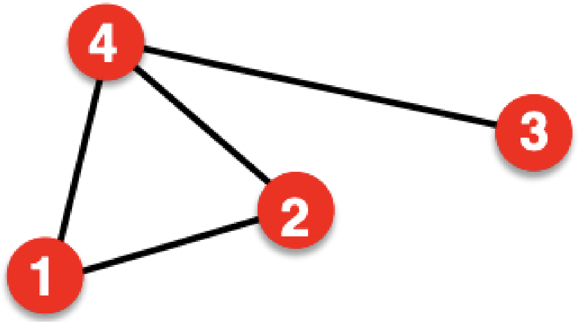
Adjacency matrix



Symmetric

$$\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

Encoder-decoder for graph data

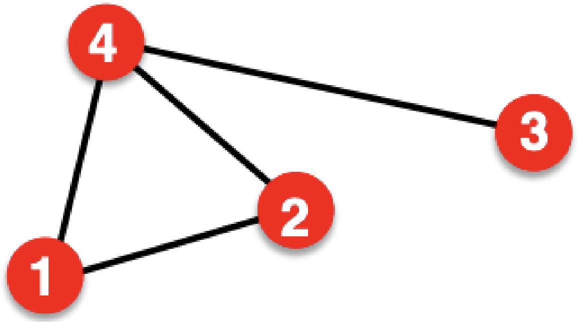


V: {1, 2, 3, 4}

$$A = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

Image credit http://web.stanford.edu/class/cs224w/slides/03-nodeemb.pdf

Encoder-decoder for graph data



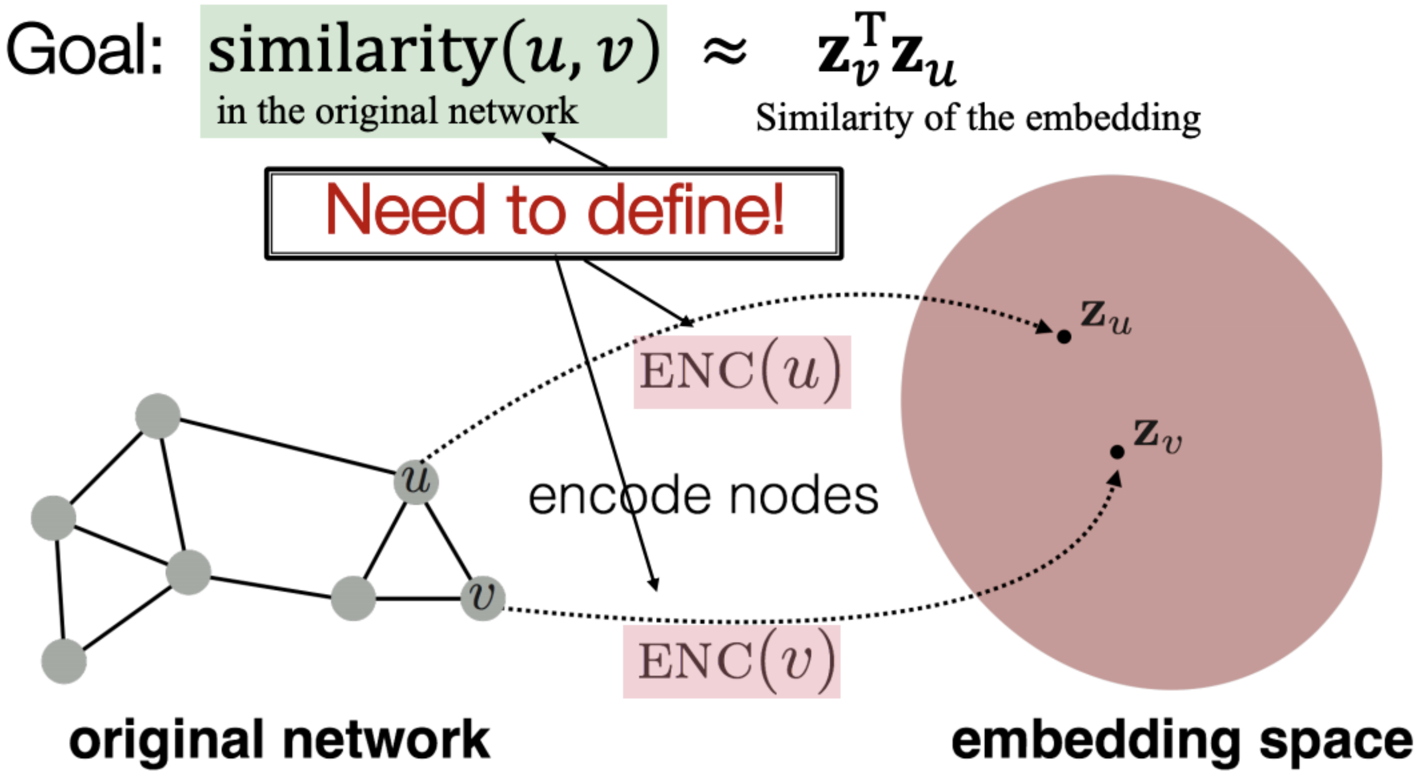
V: {1, 2, 3, 4}

$$A = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

Q: can we learn node features with the correlation in the adjacency matrix?

Image credit http://web.stanford.edu/class/cs224w/slides/03-nodeemb.pdf

Encoder-decoder for graph data



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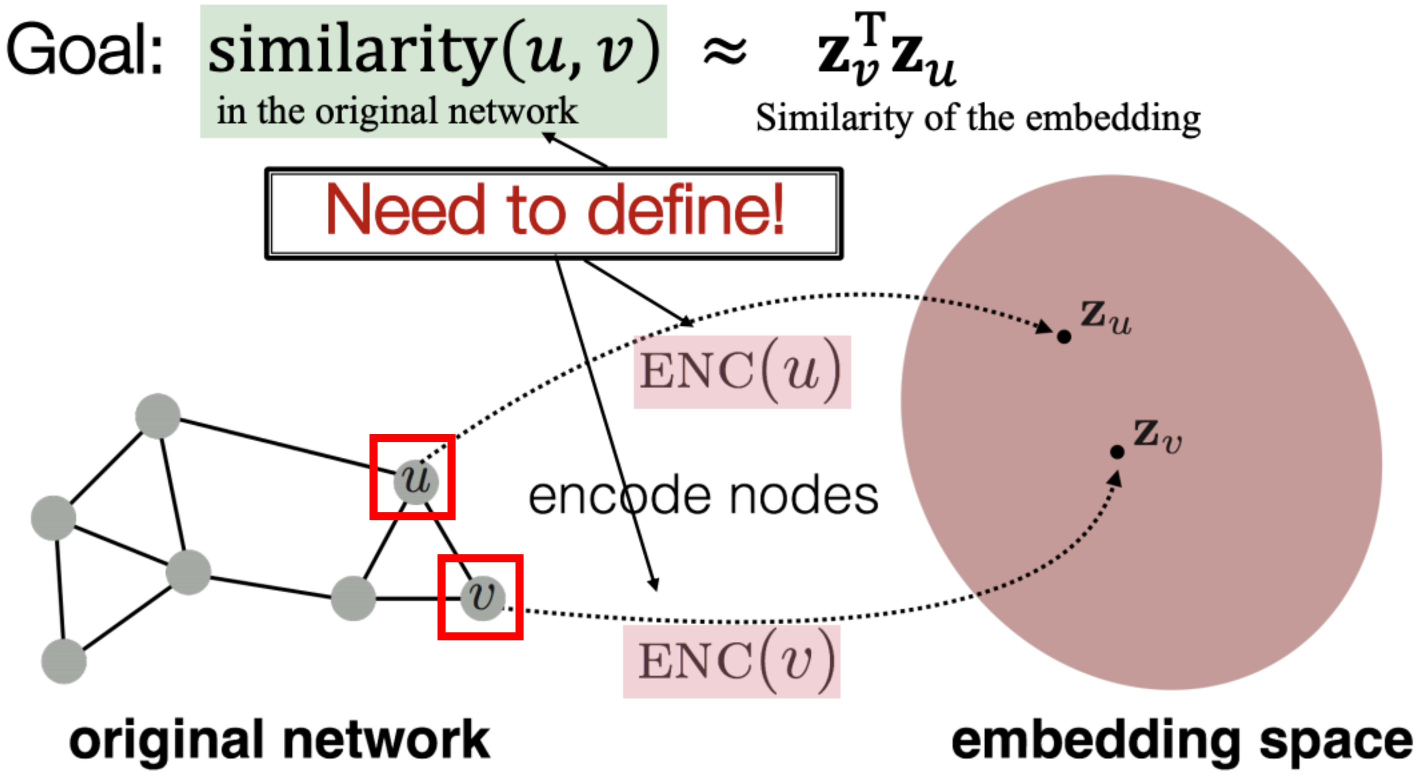
Encoder-decoder for graph data



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Encoder-decoder for graph data

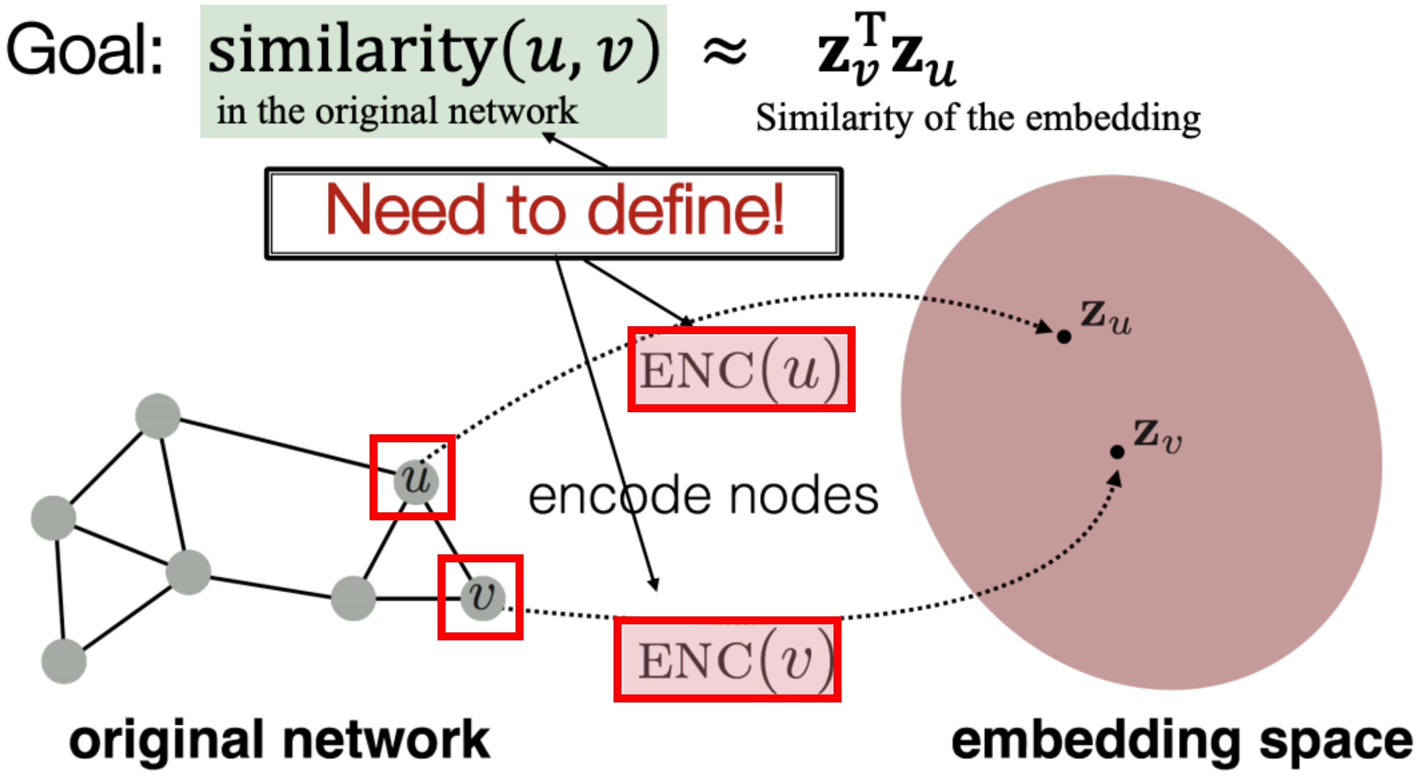
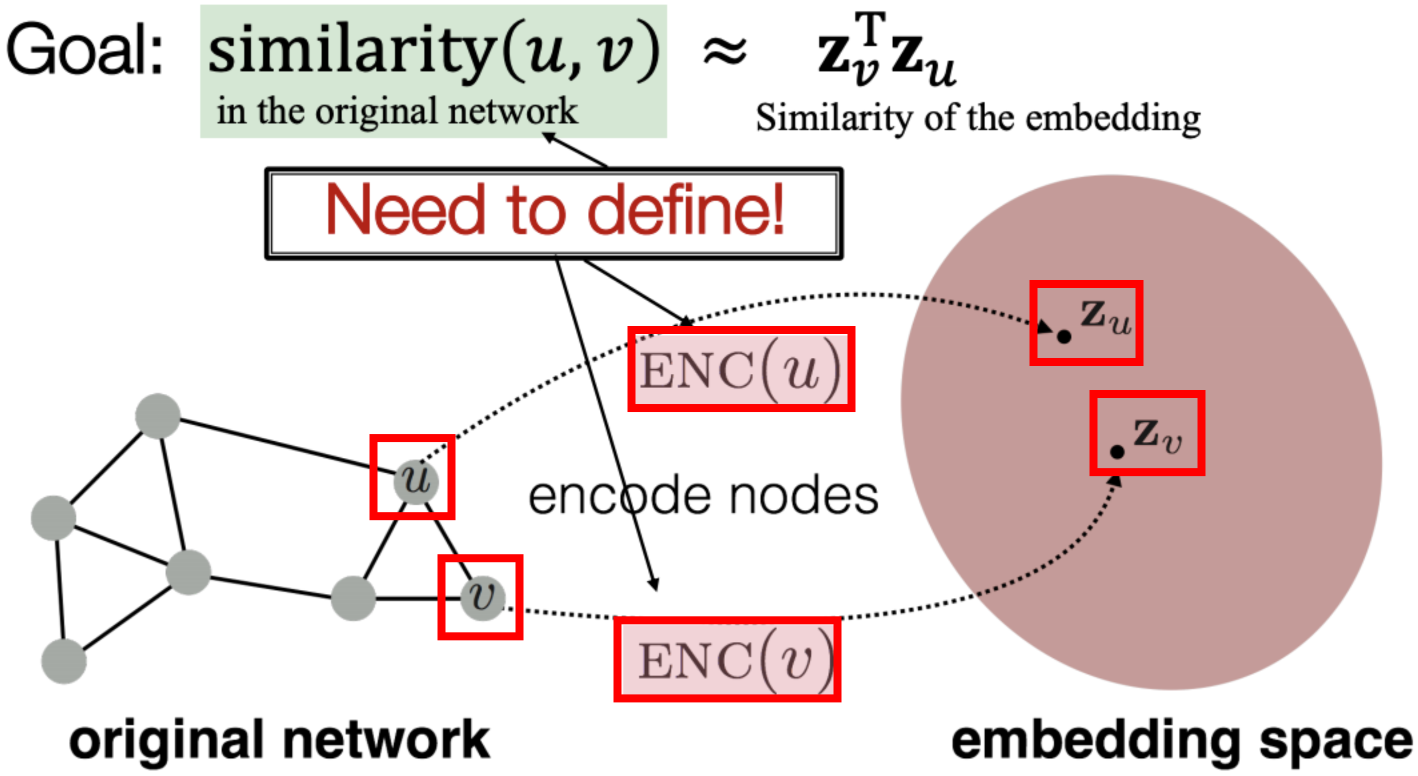


Image credit http://web.stanford.edu/class/cs224w/slides/03-nodeemb.pdf

Encoder-decoder for graph data



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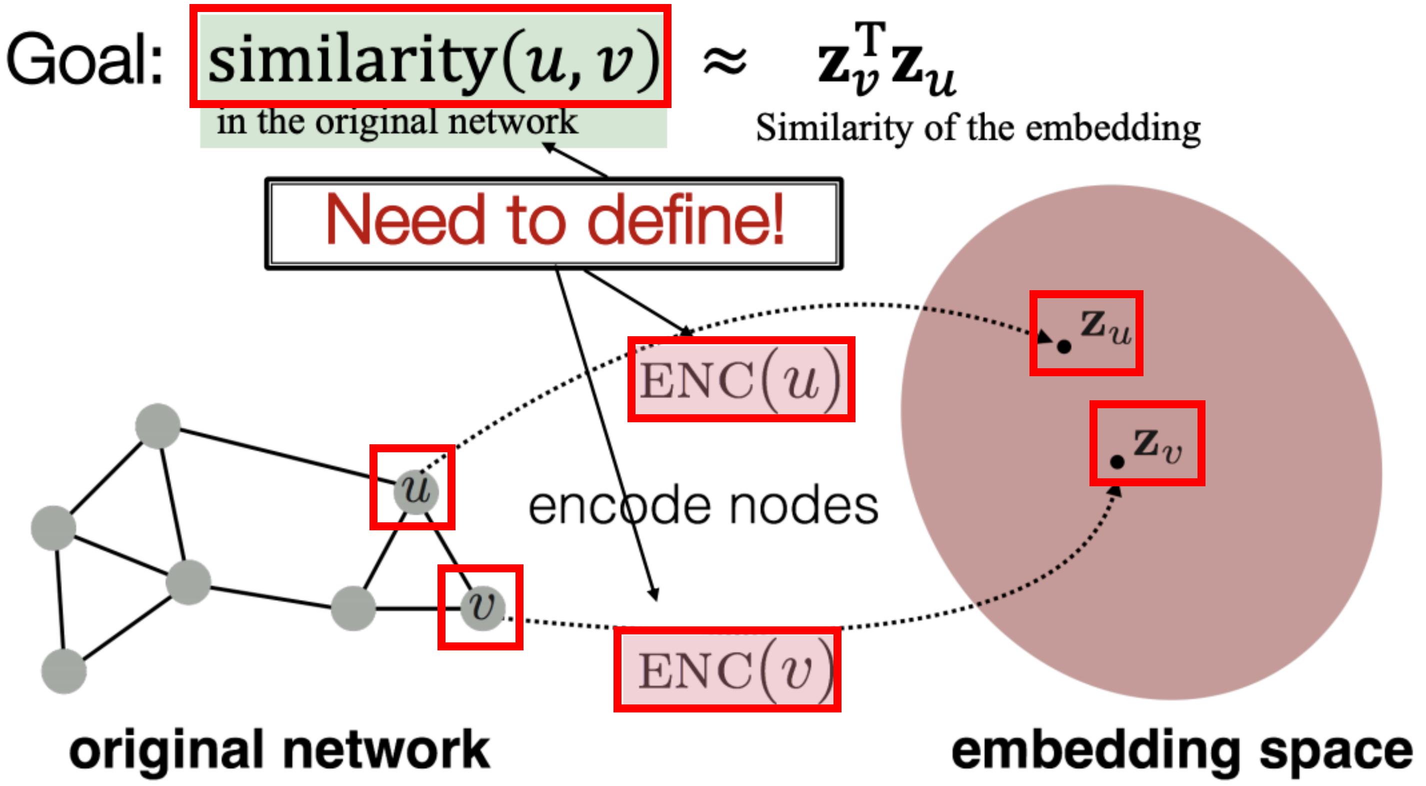

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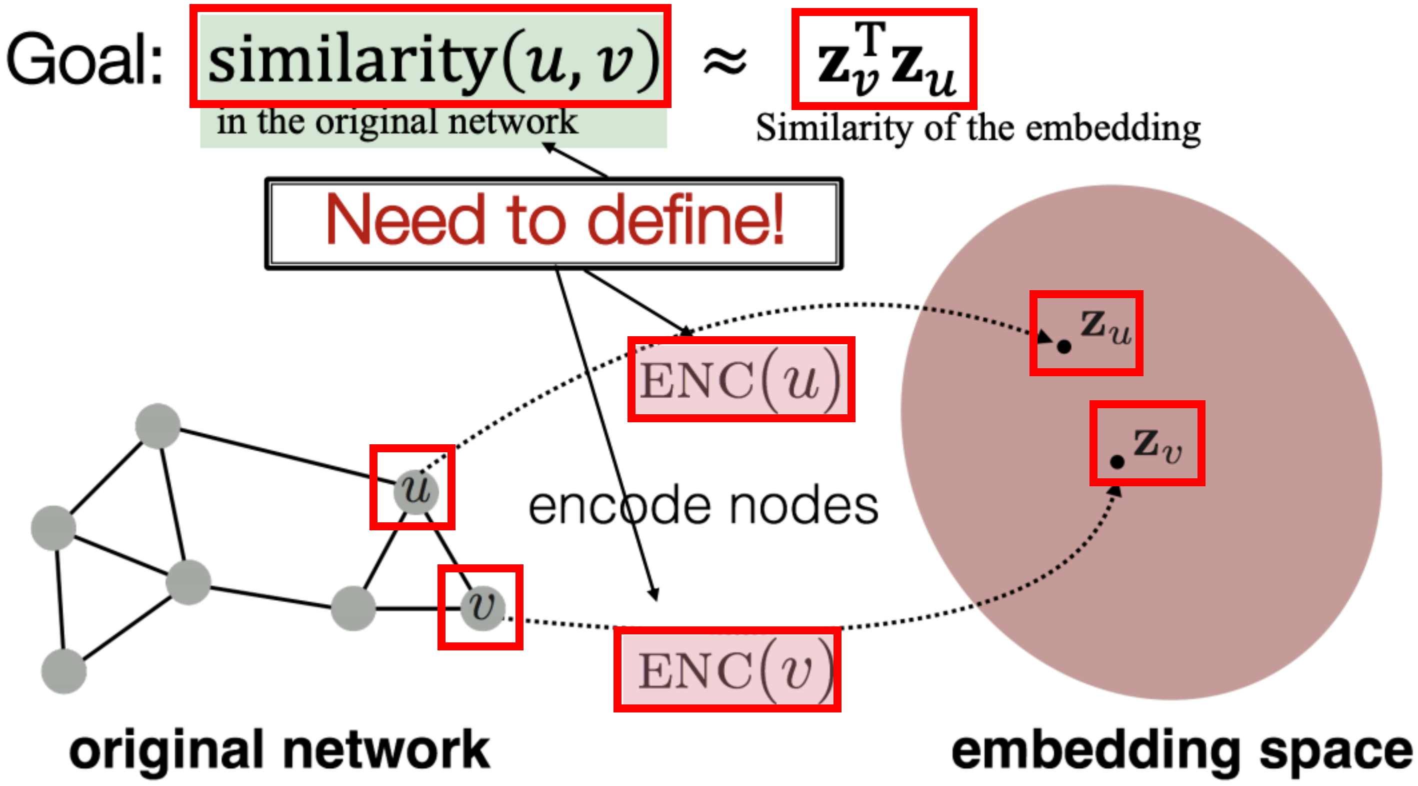

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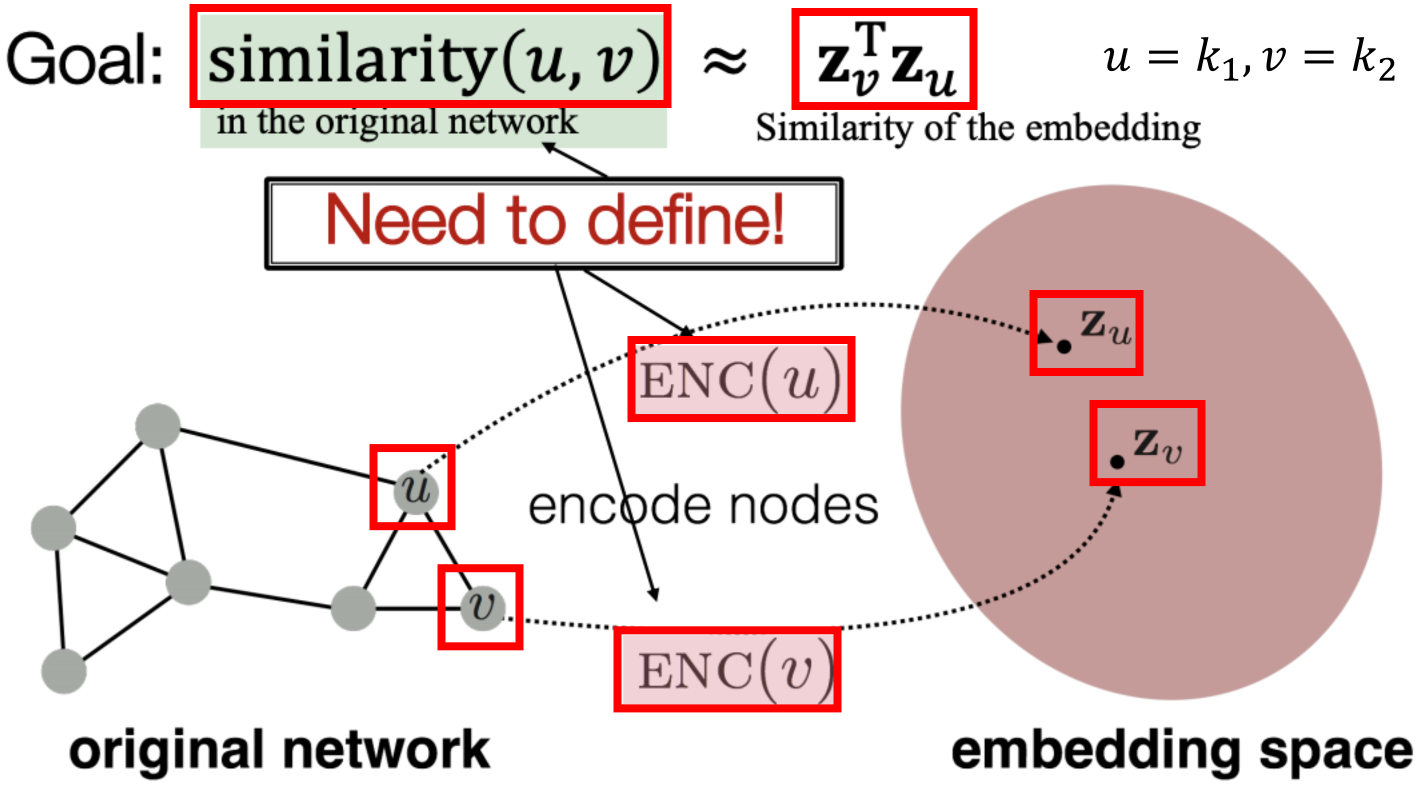

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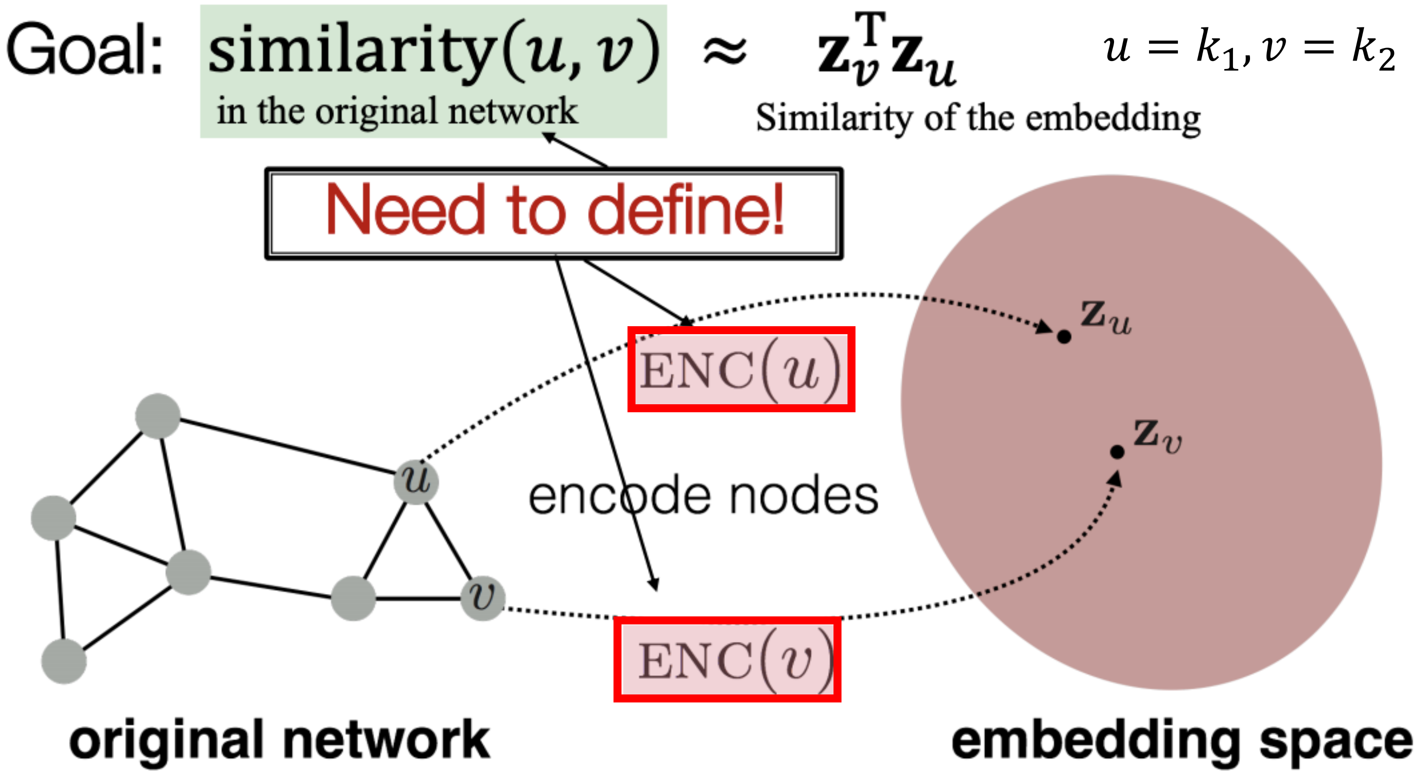

$u = k_1, v = k_2$

Q: how to learn ENC?

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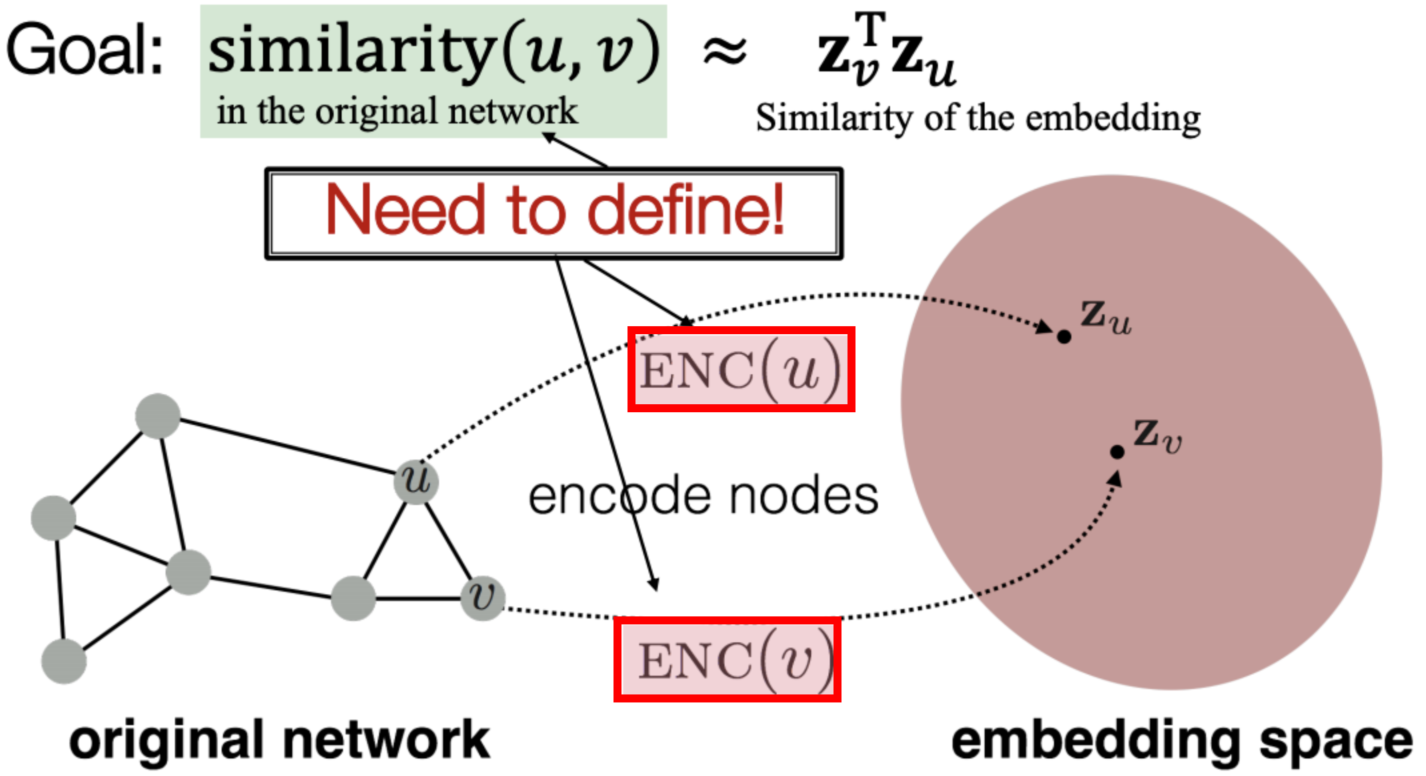
Encoder-decoder for graph data



Q: how to learn ENC?

$$\boxed{\mathrm{ENC}(v)} = \mathbf{z}_v = \mathbf{Z} \cdot v$$

$\mathbf{Z} \in \mathbb{R}^{d \times |\mathcal{V}|}$ matrix, each column is a node embedding [what we learn / optimize]

$v \in \mathbb{I}^{|\mathcal{V}|}$ indicator vector, all zeroes except a one in column indicating node *v*

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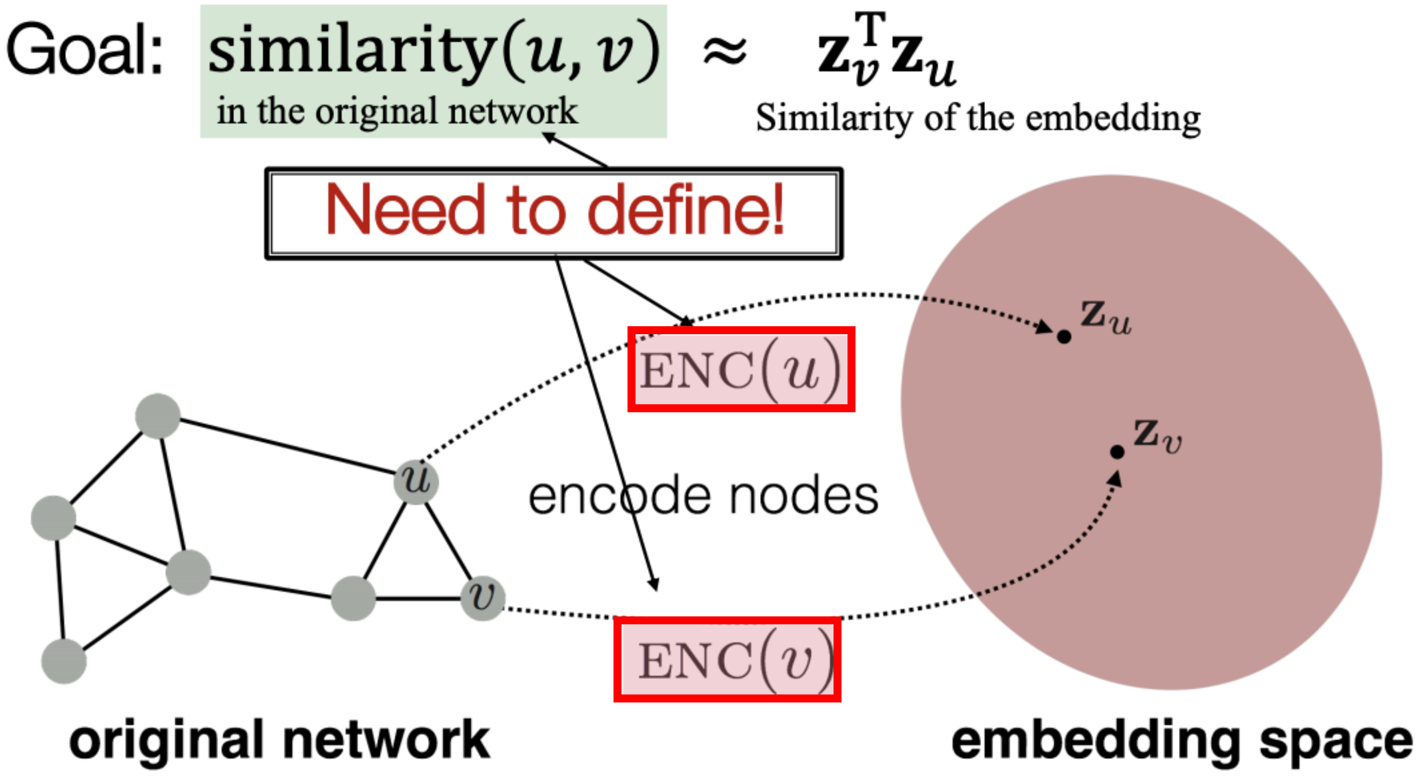
Encoder-decoder for graph data



Q: how to learn ENC?
Linear transformation

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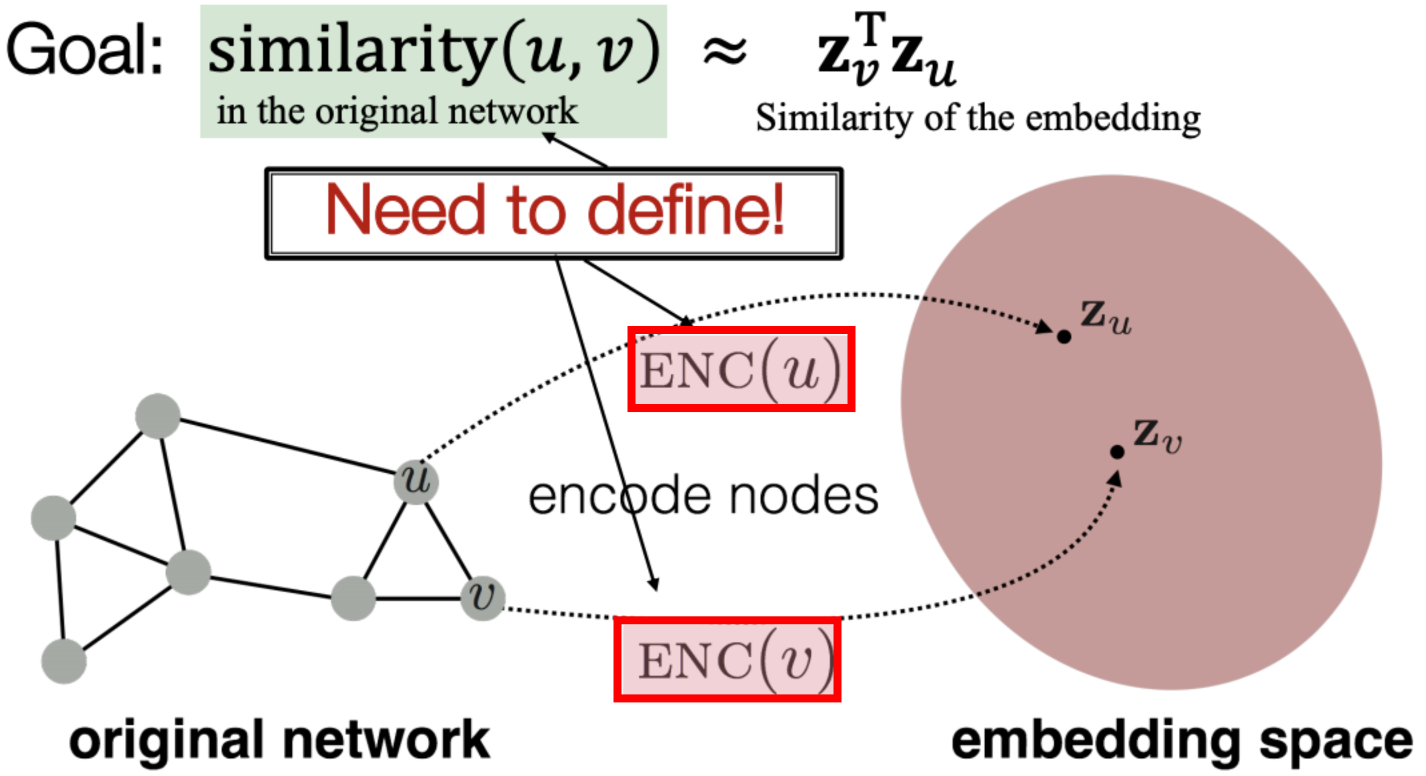

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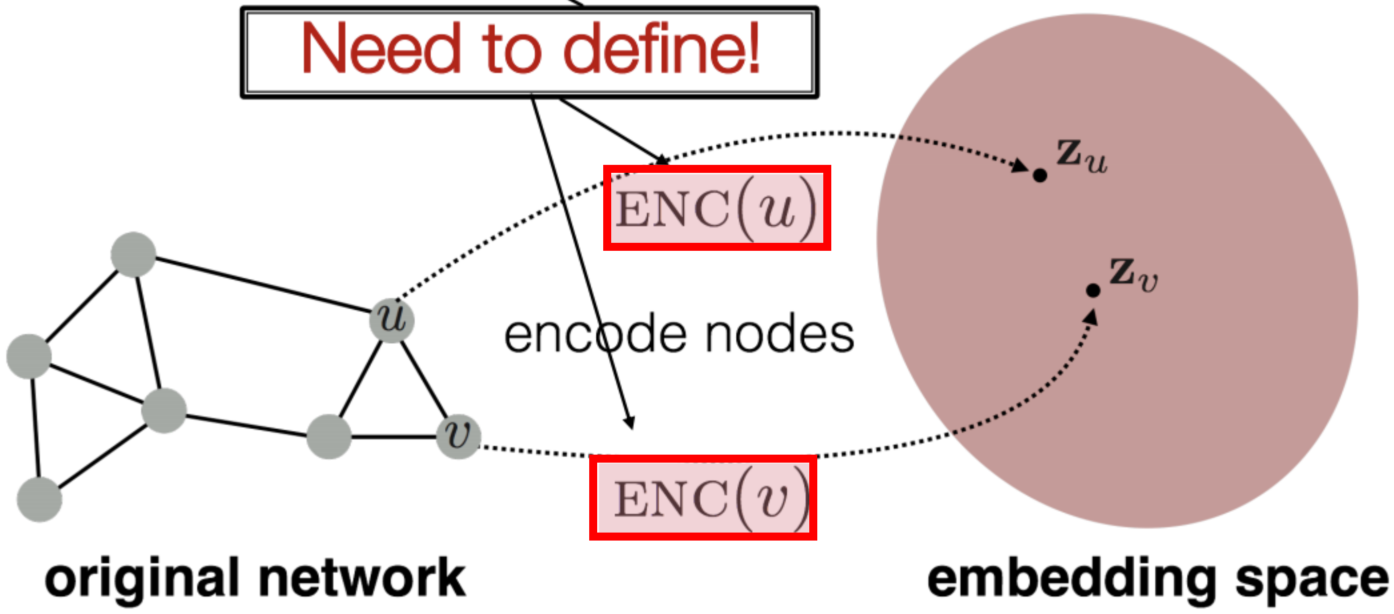
Encoder-decoder for graph data

Goal: $\underbrace{\text{similarity}(u, v)}_{\text{in the original network}} \approx \underbrace{\mathbf{z}_v^{\mathrm{T}} \mathbf{z}_u}_{\text{Similarity of the embedding}}$



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Linear transformation

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One-hot labels

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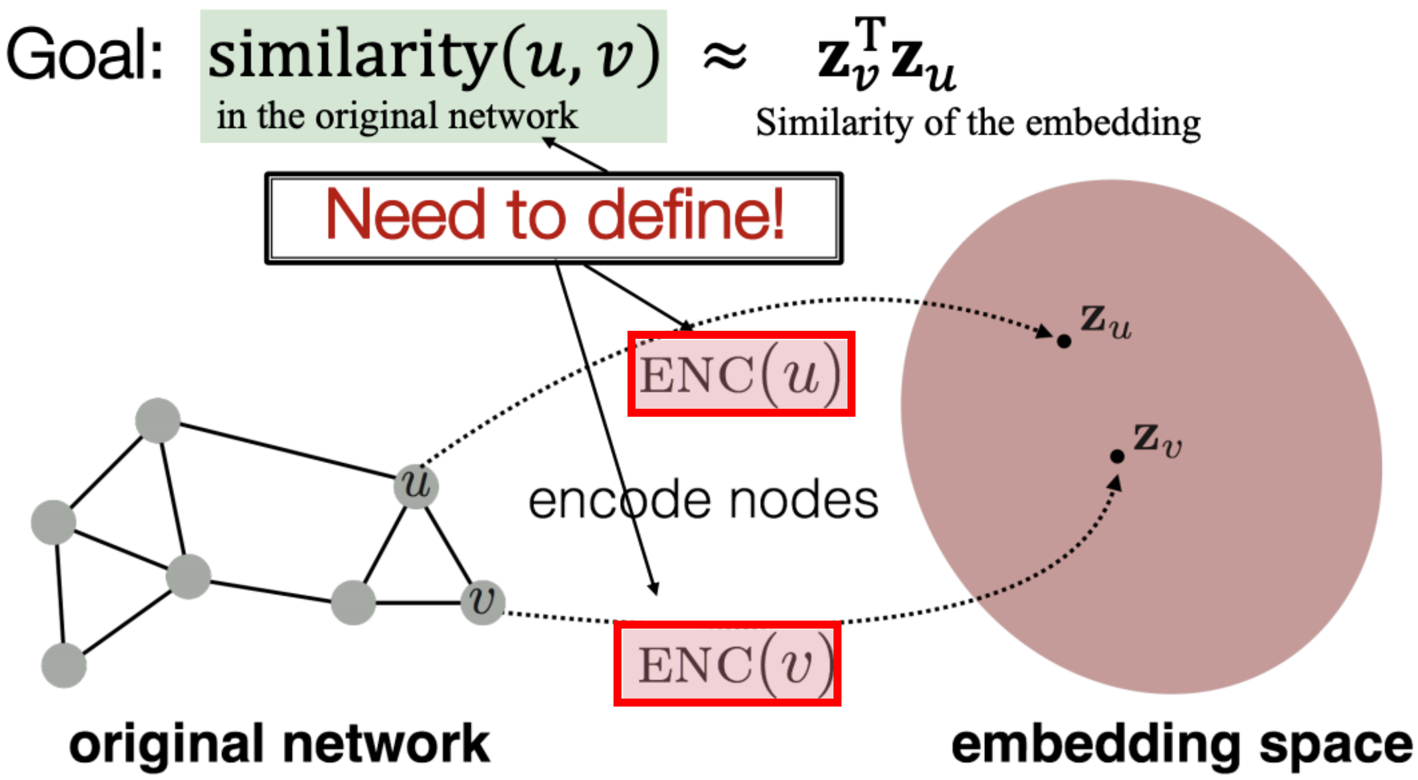
Encoder-decoder for graph data



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$\boxed{\mathbf{Z}} \in \mathbb{R}^{d \times |\mathcal{V}|}$

$\boxed{v} \in \mathbb{I}^{|\mathcal{V}|}$

Binary	Gray code	One–hot
000	000	00000001
001	001	00000010
010	011	00000100
011	010	00001000
100	110	00010000
101	111	00100000
110	101	01000000
111	100	10000000

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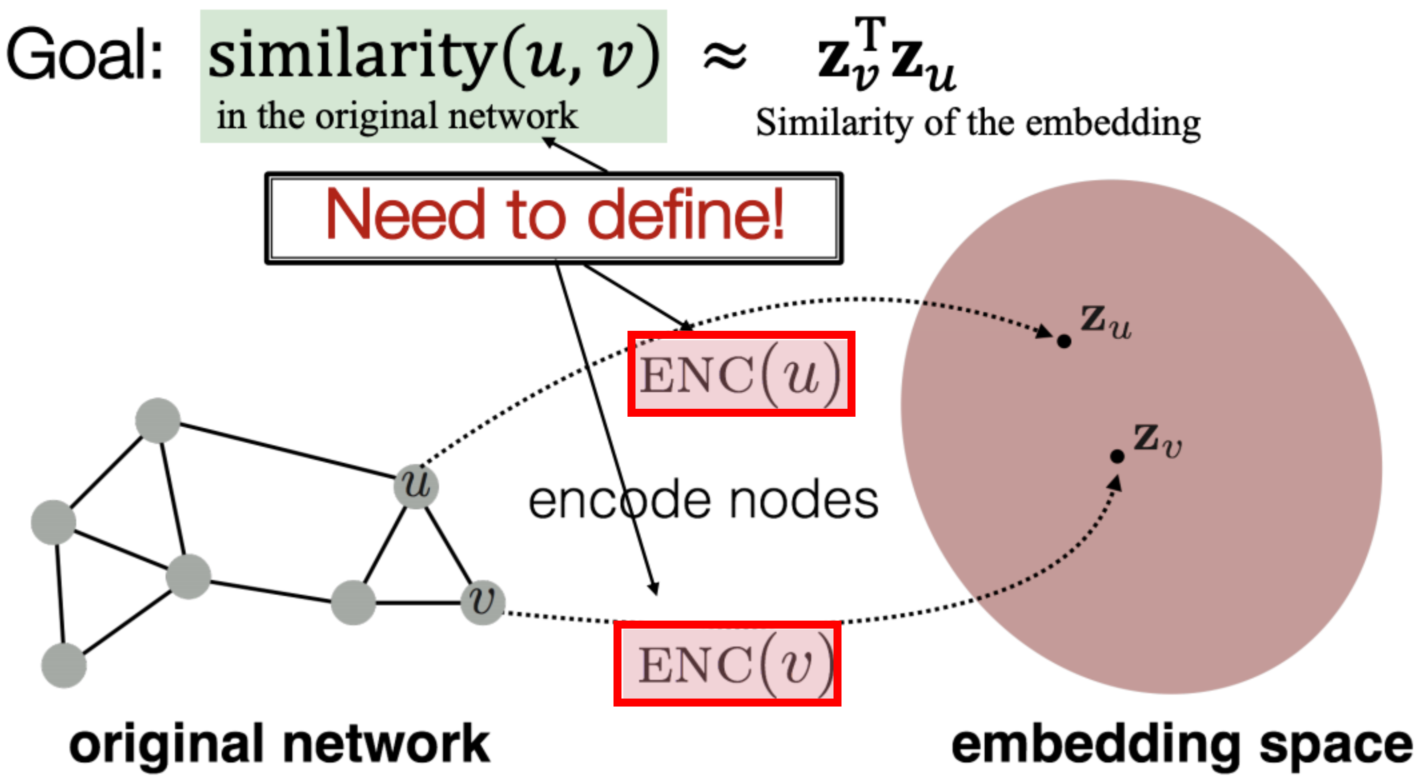
Encoder-decoder for graph data



Q: how to learn ENC?
Linear transformation

$\boxed{\mathrm{ENC}(v)} = \mathbf{z}_v = \mathbf{Z} \cdot v$

$\boxed{\mathbf{Z}} \in \mathbb{R}^{d\times|\mathcal{V}|}$

$\boxed{v} \in \mathbb{I}^{|\mathcal{V}|}$

Each node

Binary	Gray code	One-hot
000	000	00000001
001	001	00000010
010	011	00000100
011	010	00001000
100	110	00010000
101	111	00100000
110	101	01000000
111	100	10000000

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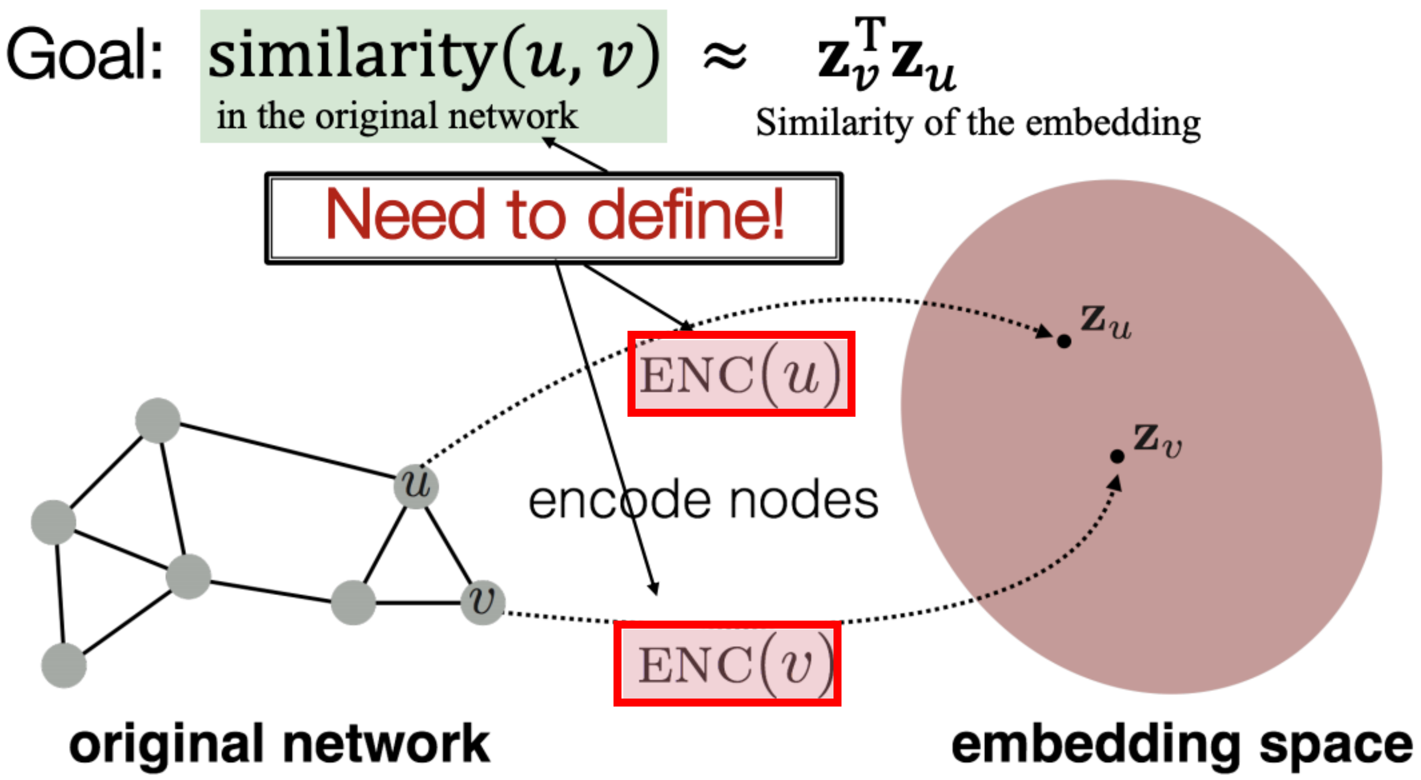
Encoder-decoder for graph data



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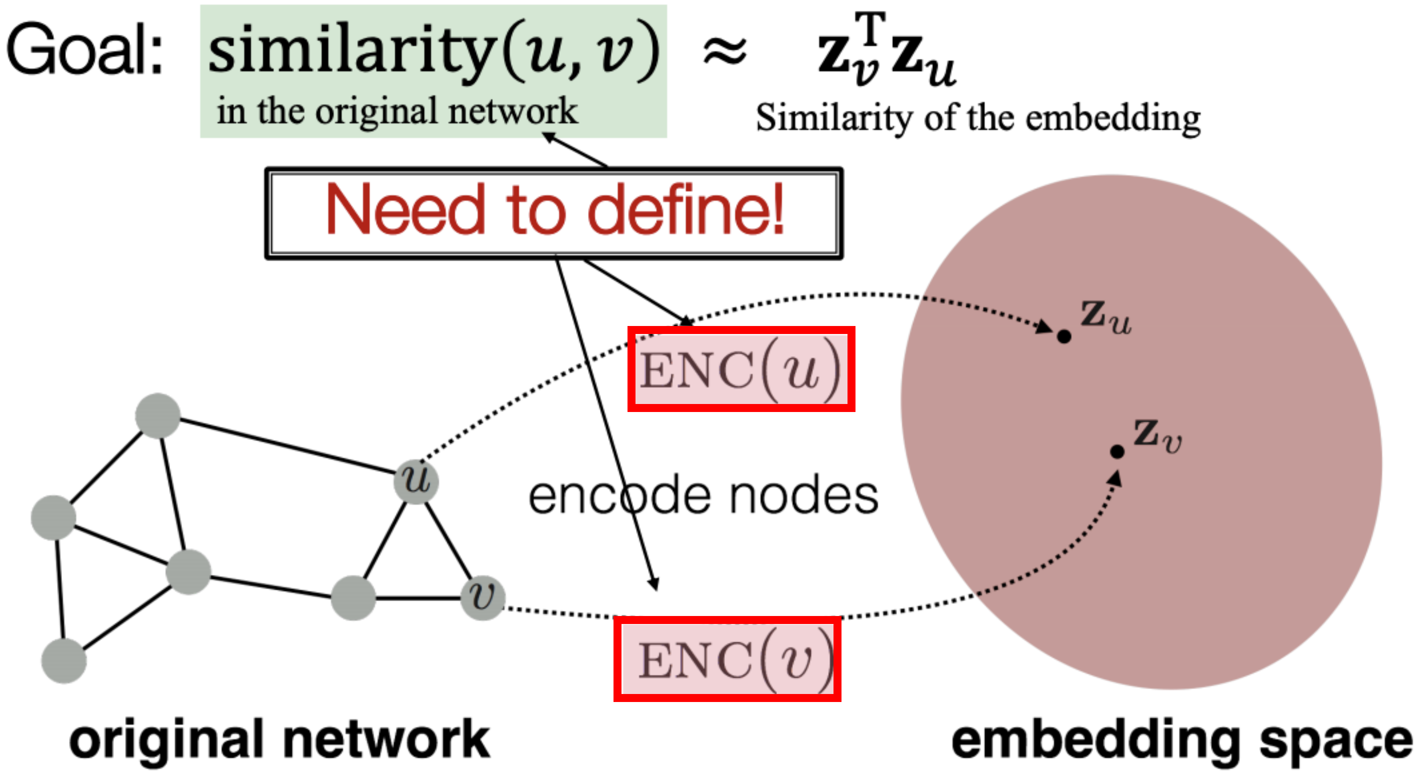
Encoder-decoder for graph data



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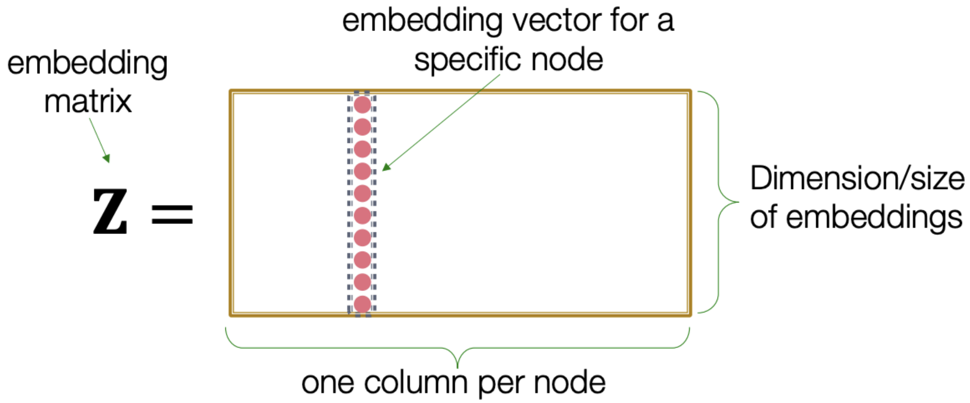

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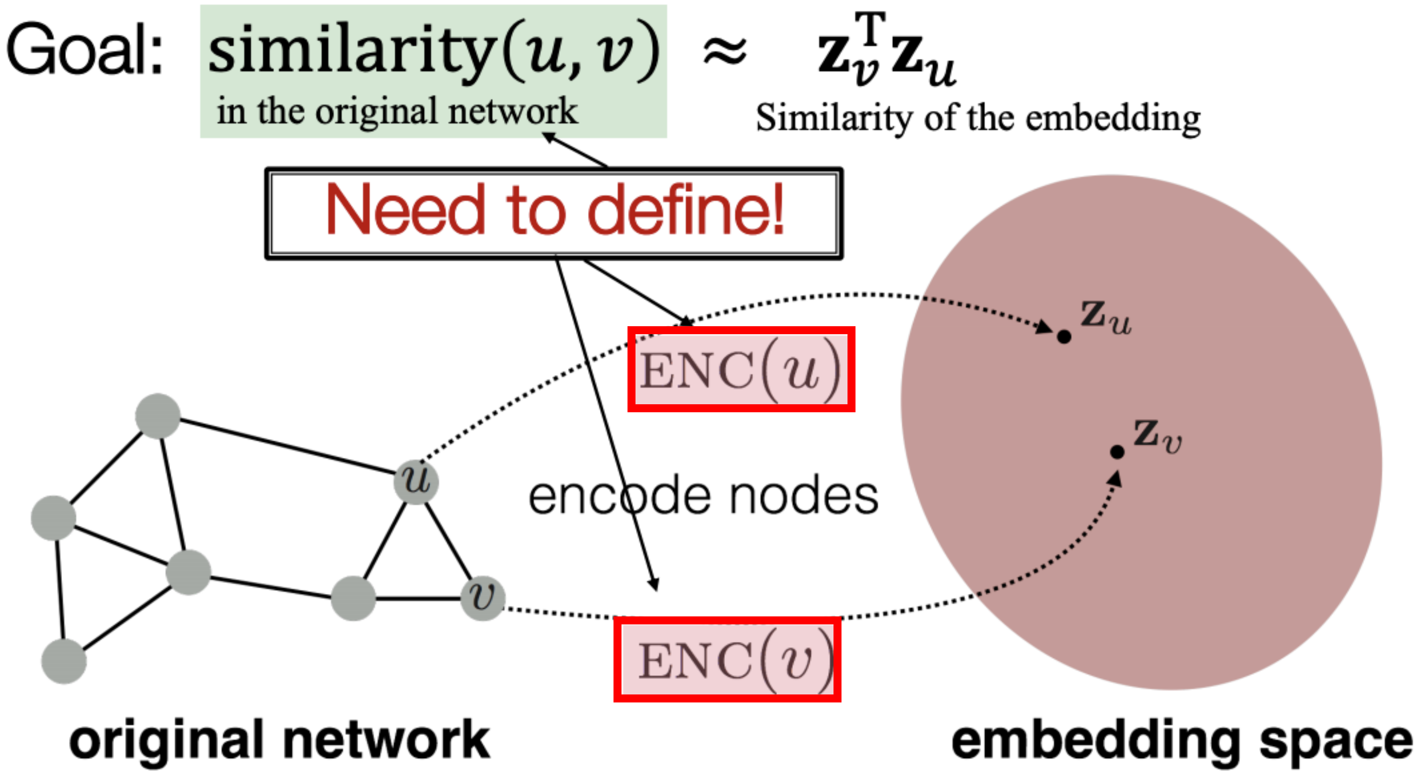
Encoder-decoder for graph data



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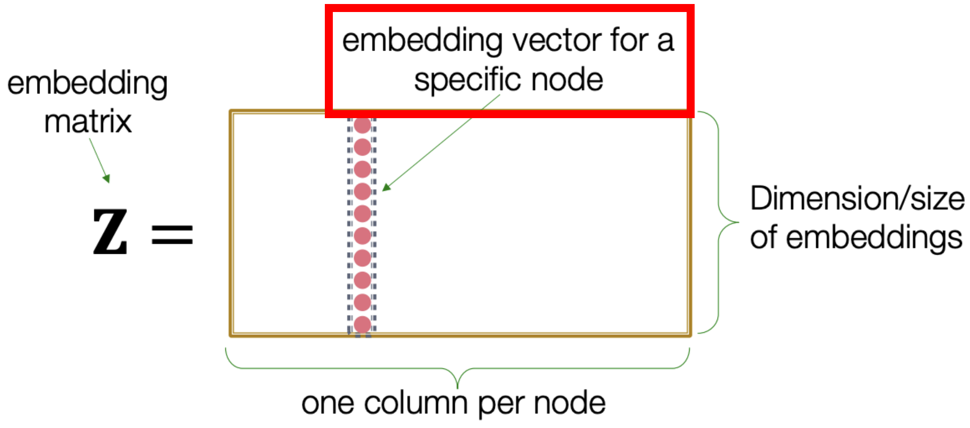

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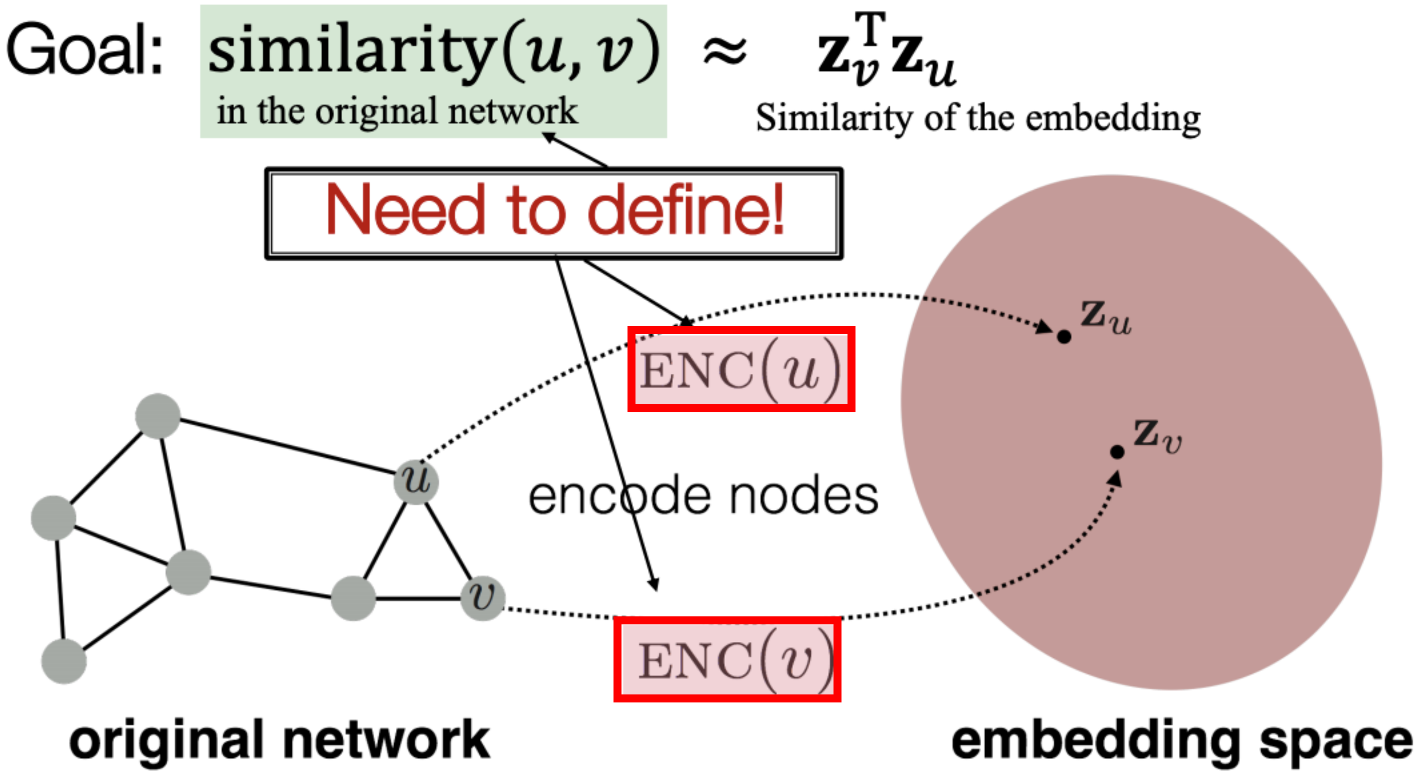
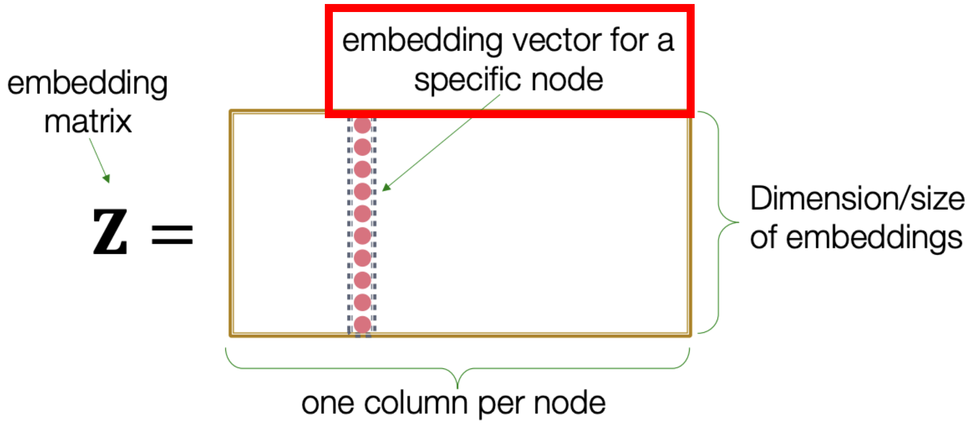
Encoder-decoder for graph data



Q: how to learn ENC?



Learnable parameters

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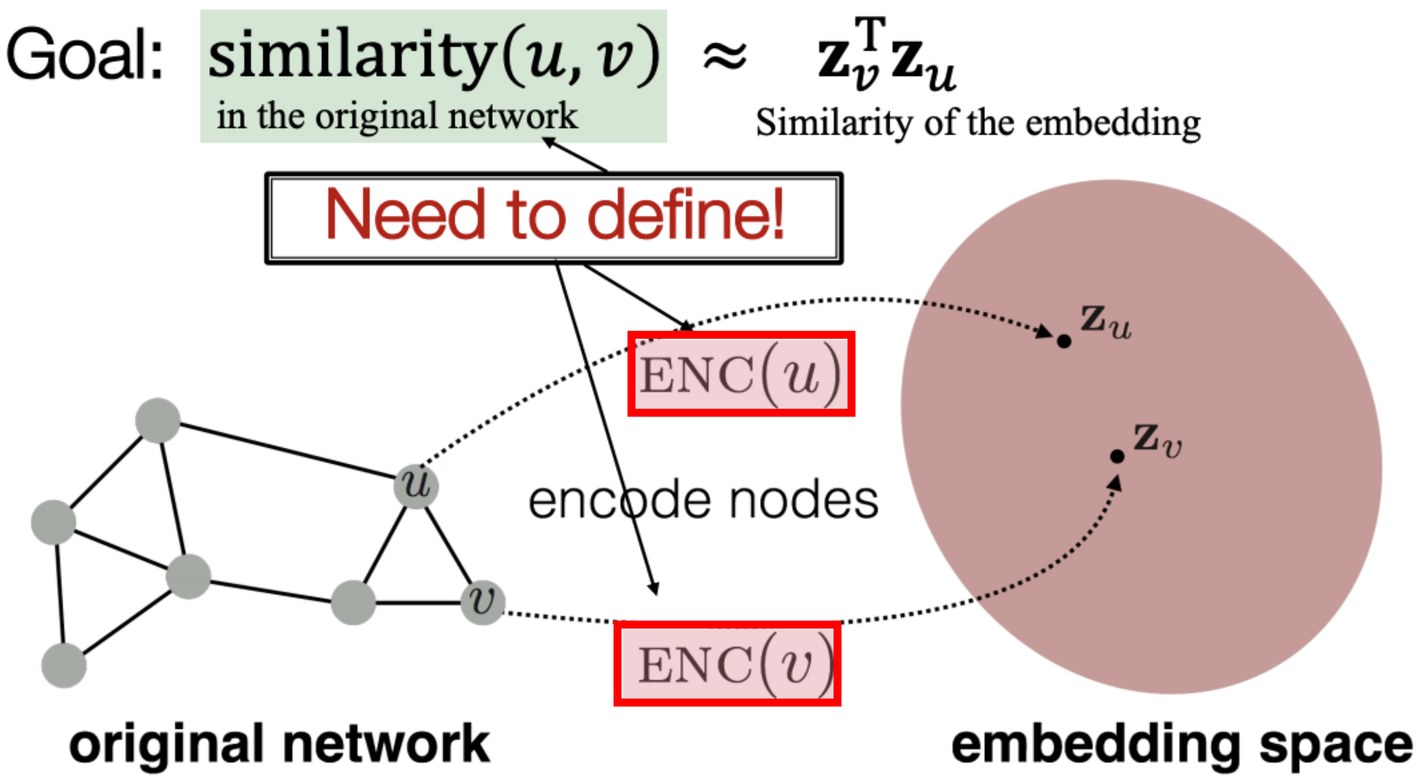
Encoder-decoder for graph data



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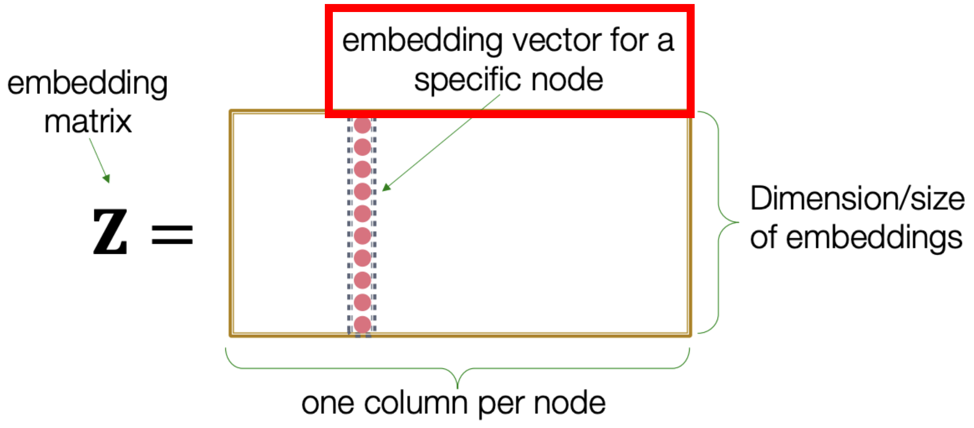

Learnable parameters

Q: how can we replace linear node embedding with nonlinear functions?

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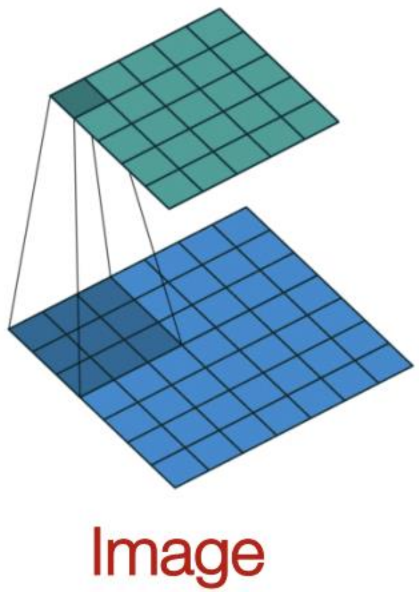
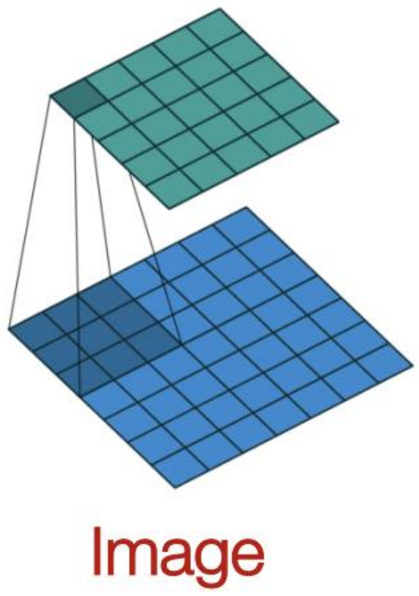
Graph neural networks



Graph neural networks



Q: can we use convolution operation on graph?

Graph convolutional neural networks