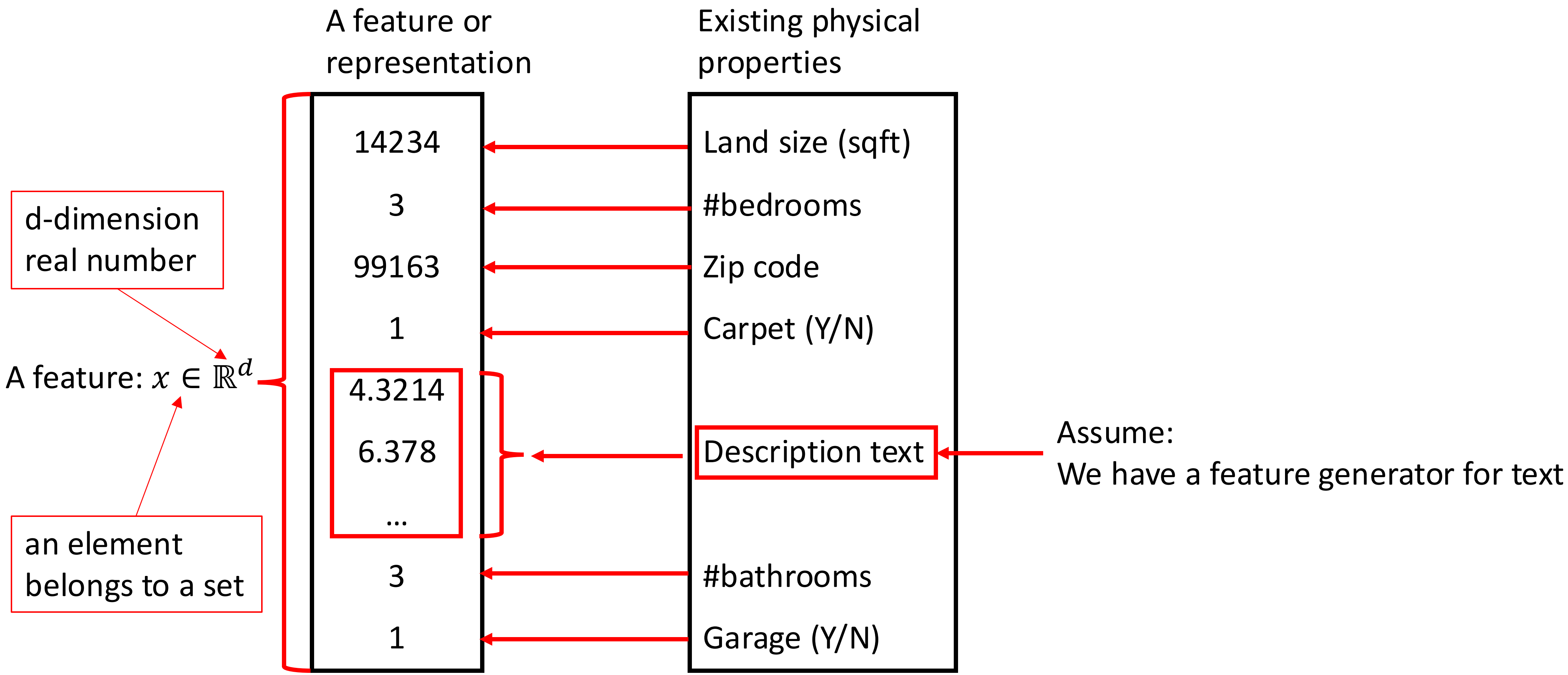
Neural Network Basics

CPT_S 434/534 Neural network design and application

Today's class includes

- Bag-of-words features (hand-crafted)
  - TF-IDF for text data
  - HOG for image data
- History of convolutional neural networks
  - Difference from conventional machine learning methods such as linear models (from the viewpoint of feature generation)
- Feedforward networks: a simple kind of neural networks
  - Typical structure, properties and examples

House price prediction



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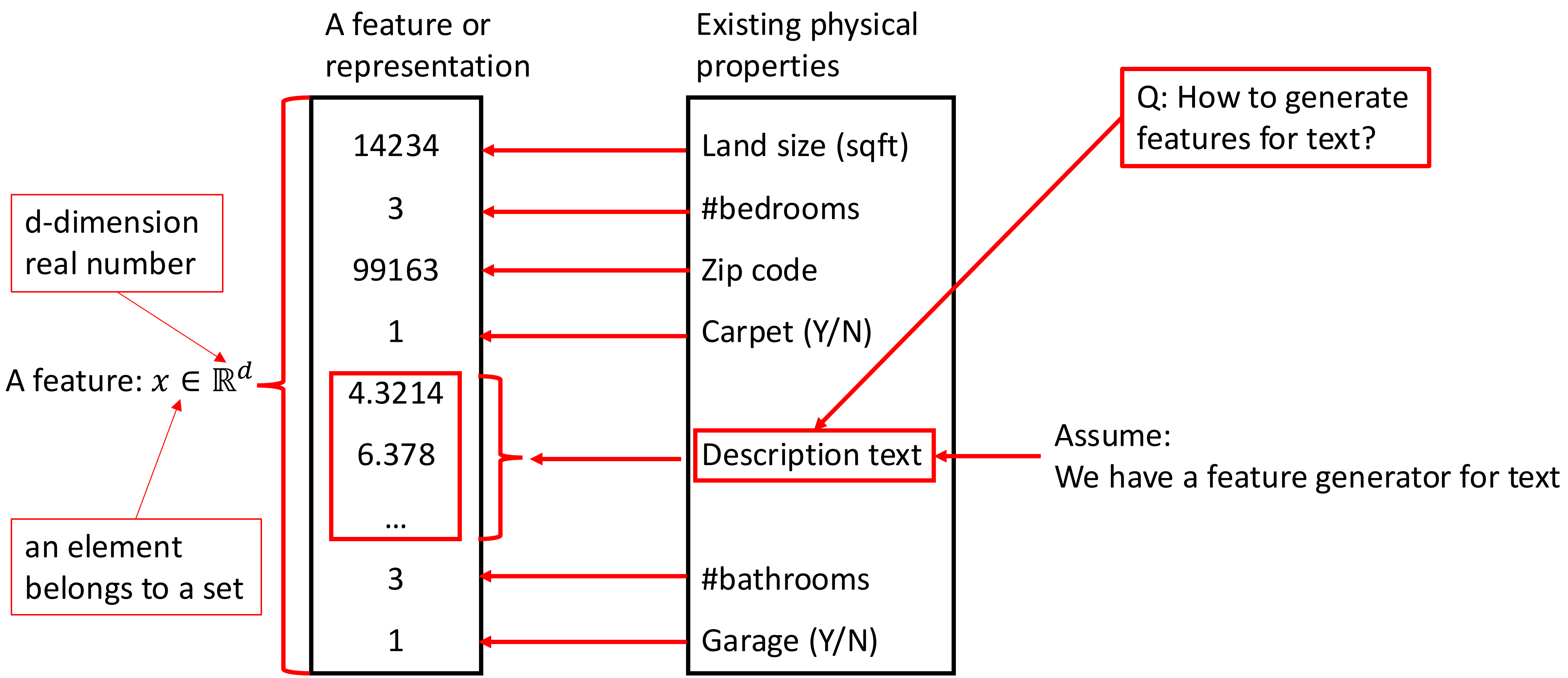

Bag-of-words features

- TF-IDF (term frequency–inverse document frequency)

t: a term
d: a document
D: a set of documents

$$\mathrm{tfidf}(t, d, D) = \mathrm{tf}(t, d) \cdot \mathrm{idf}(t, D)$$

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total number of documents in the **corpus D** (i.e., |D|)
(A set of documents)

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(seen as a coefficient)

Bag-of-words features

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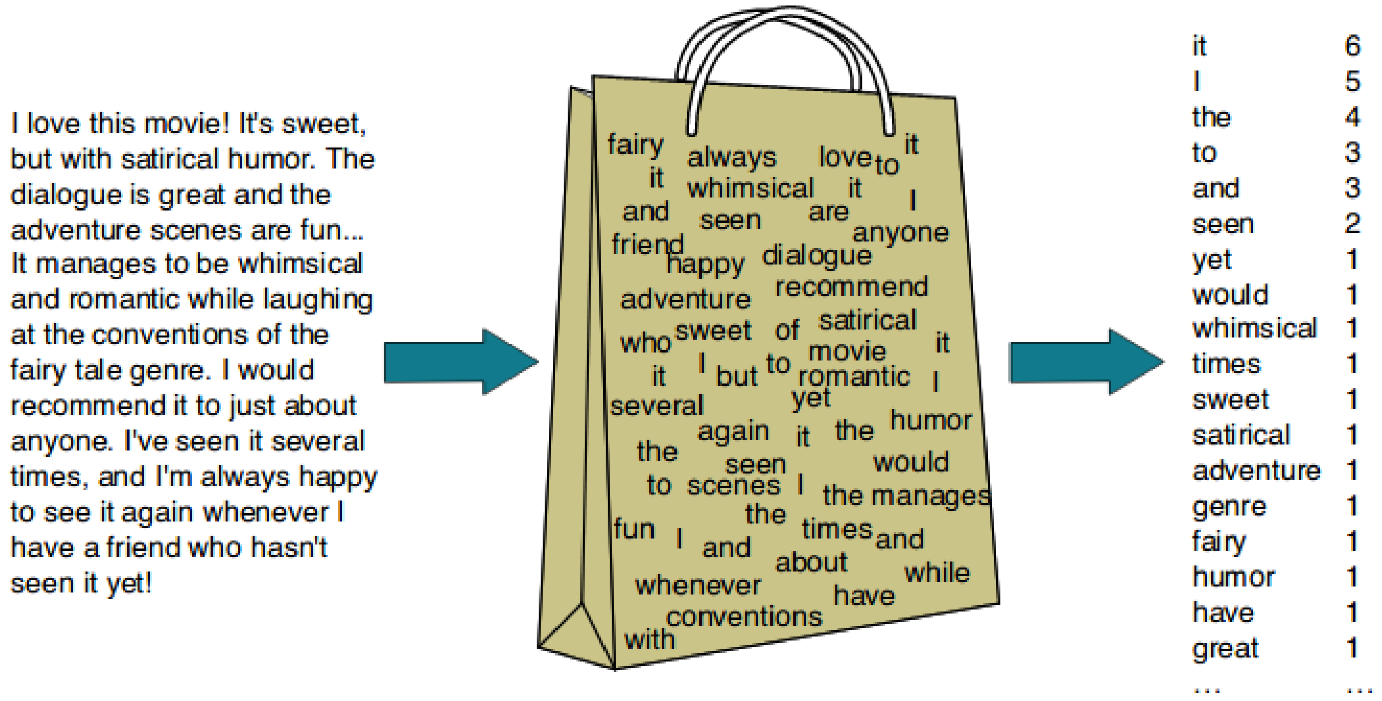

Image from https://heartbeat.fritz.ai/spam-filtering-using-bag-of-words-1c5484ff07f1

Bag-of-words features

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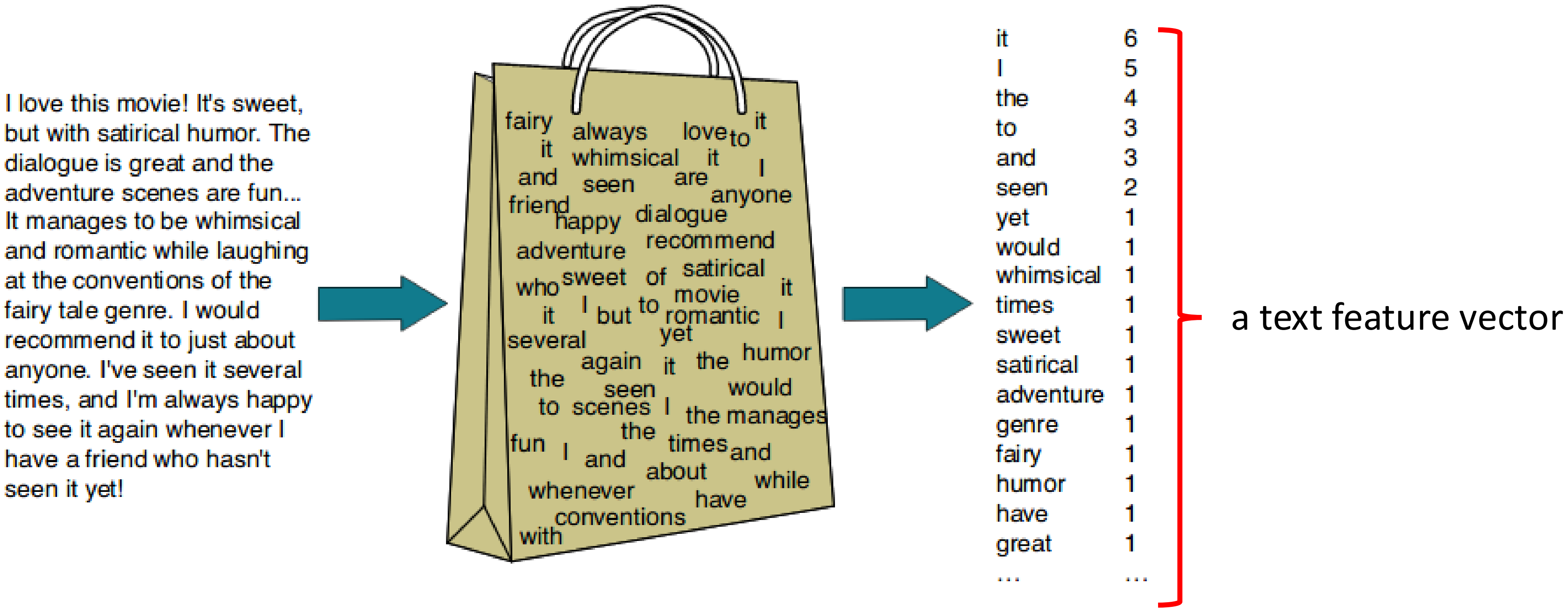

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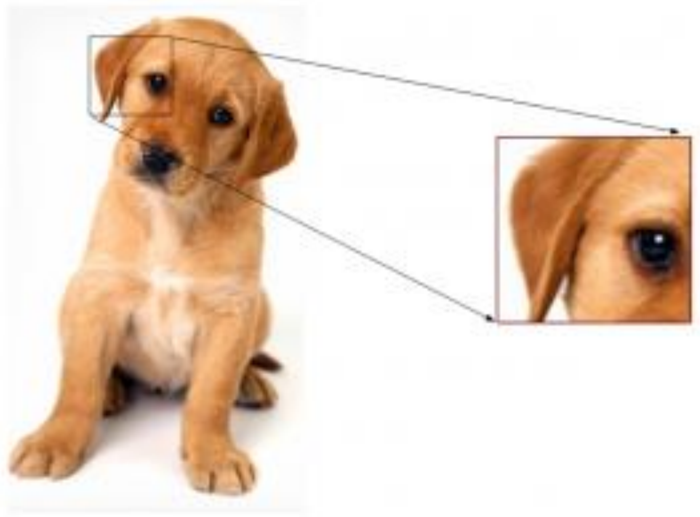
Bag-of-words features

Q: How to generate a bag-of-words feature for an image?

Histogram of oriented gradients

- Oriented gradients?
  - Gradients: changes in X and Y directions



Credit for https://www.analyticsvidhya.com/blog/2019/09/feature-engineering-images-introduction-hog-feature-descriptor/

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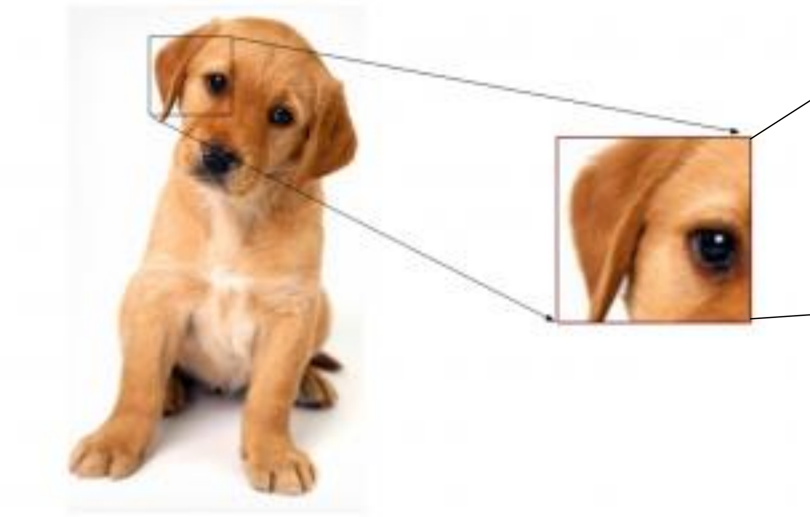

Pixel values in the image

121	10	78	96	125
48	152	68	125	111
145	78	85	89	65
154	214	56	200	66
214	87	45	102	45

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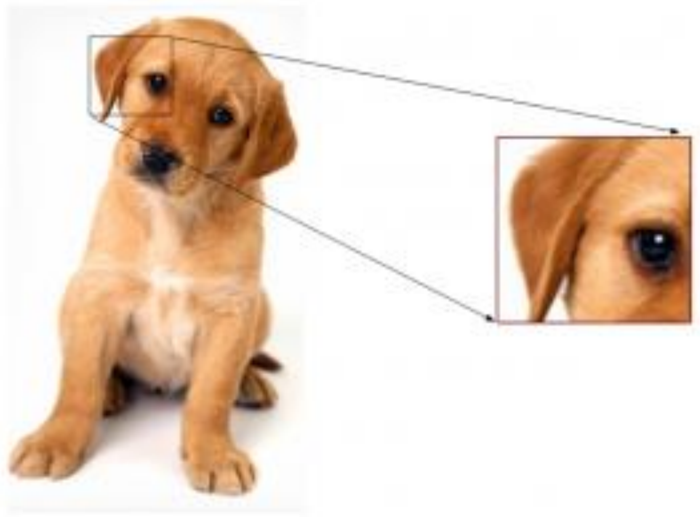

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X direction G_x
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G_x = 89-78 = 11

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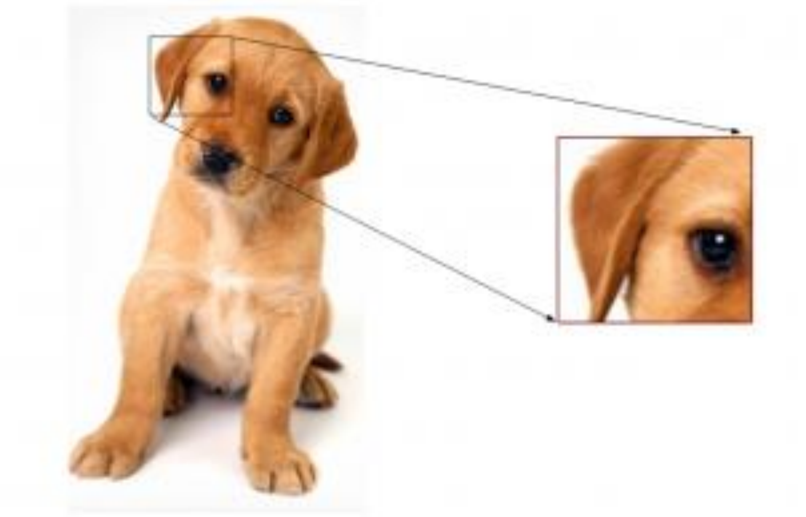

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Y direction G_y
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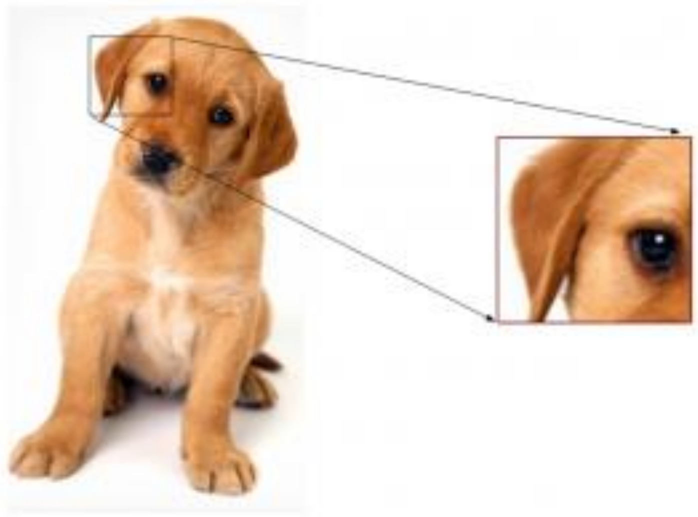

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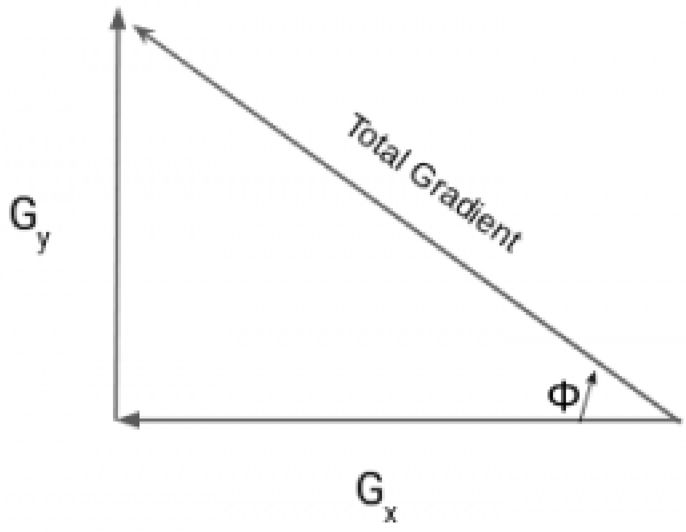

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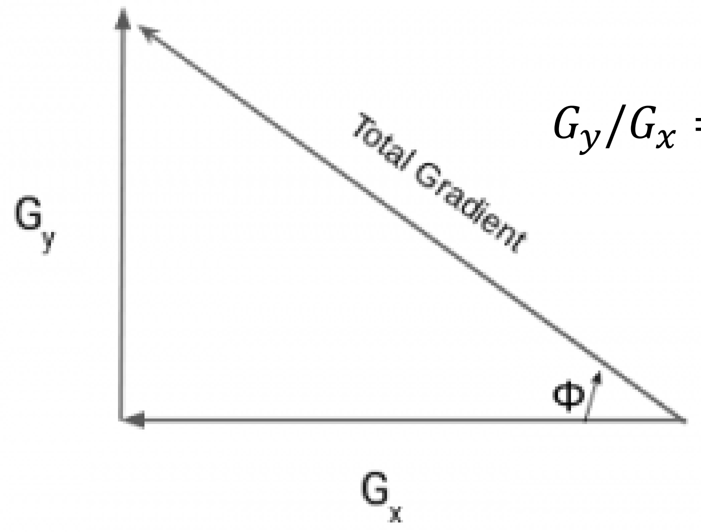

$G_y/G_x = \tan(\Phi)$

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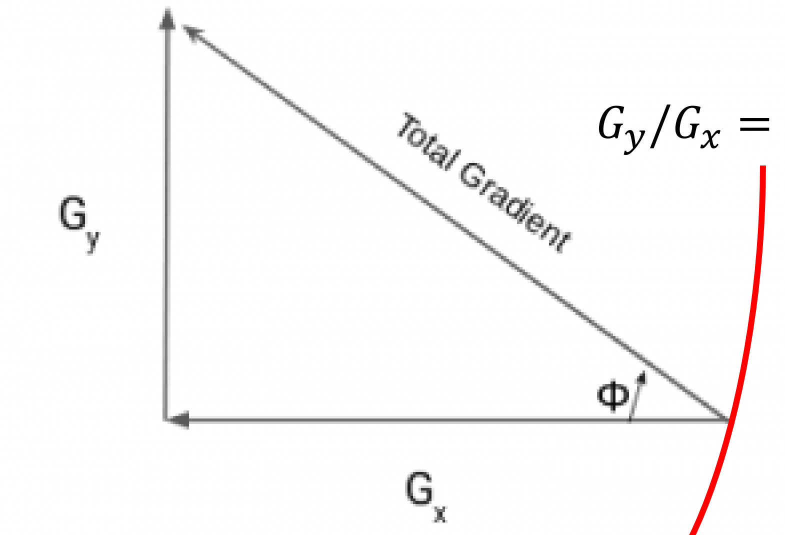

$G_y/G_x = \tan(\Phi)$

$\Phi = \tan^{-1}(G_y/G_x)$

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Frequency						1										
Angle (Φ)	1	2	3	4 ...	35	$\boxed{36}$	37	38	39....	175	176	177	178	179	180	

Credit for https://www.analyticsvidhya.com/blog/2019/09/feature-engineering-images-introduction-hog-feature-descriptor/

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How many pixels with the corresponding value of angle → a vector feature

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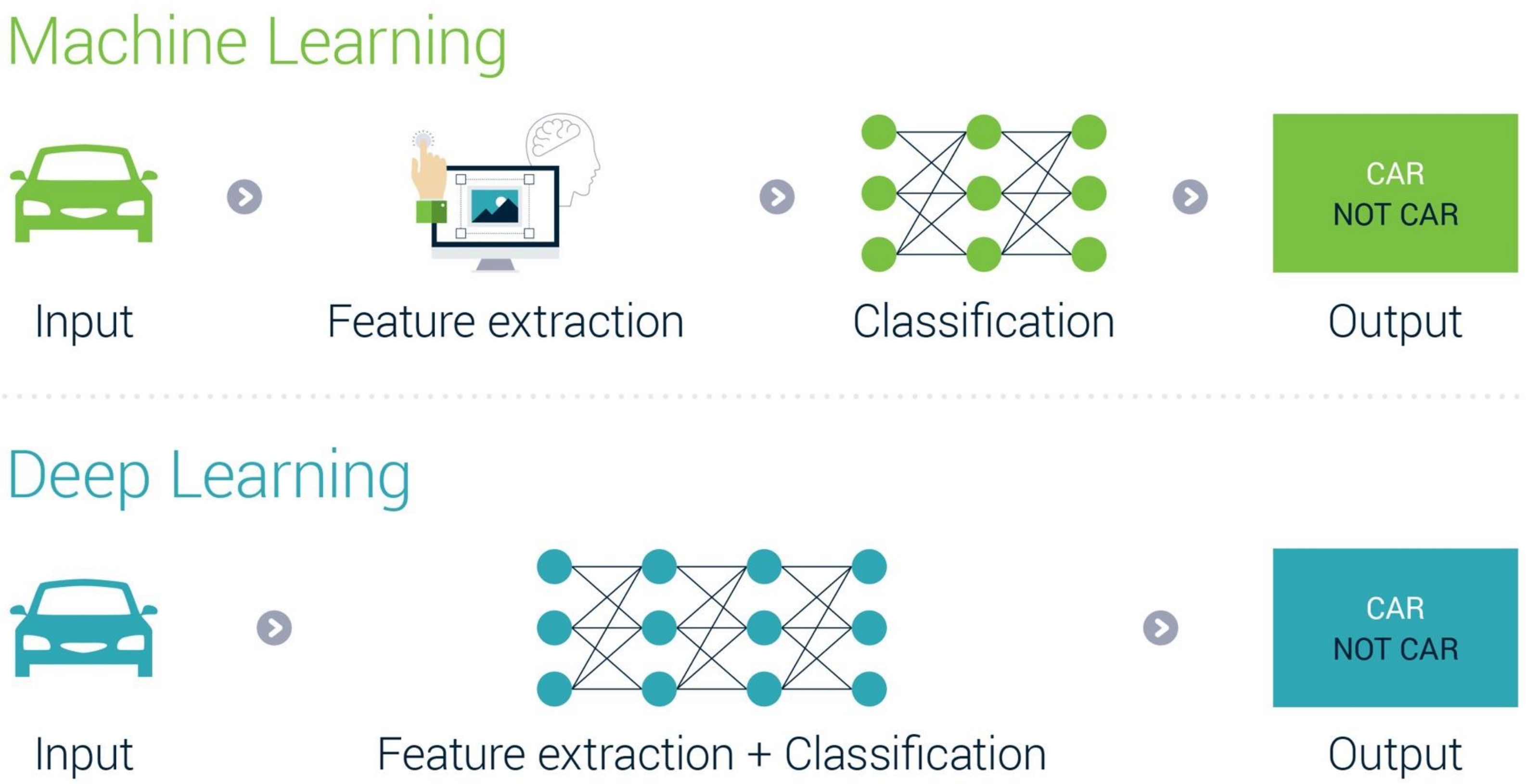
Many other hand-crafted features

- Scale Invariant Feature Transform (SIFT)
- Speeded-Up Robust Feature (SURF)
- Histogram of Optical Flow (HOF)
- Motion Boundary Histogram (MBH)
- Fisher vector (FV, a similarity-based function)
- Vector of Locally Aggregated Descriptors (VLAD)
- ……

Image BOW features

Video BOW features

End-to-end training of neural networks



LeNet-5 in 1999

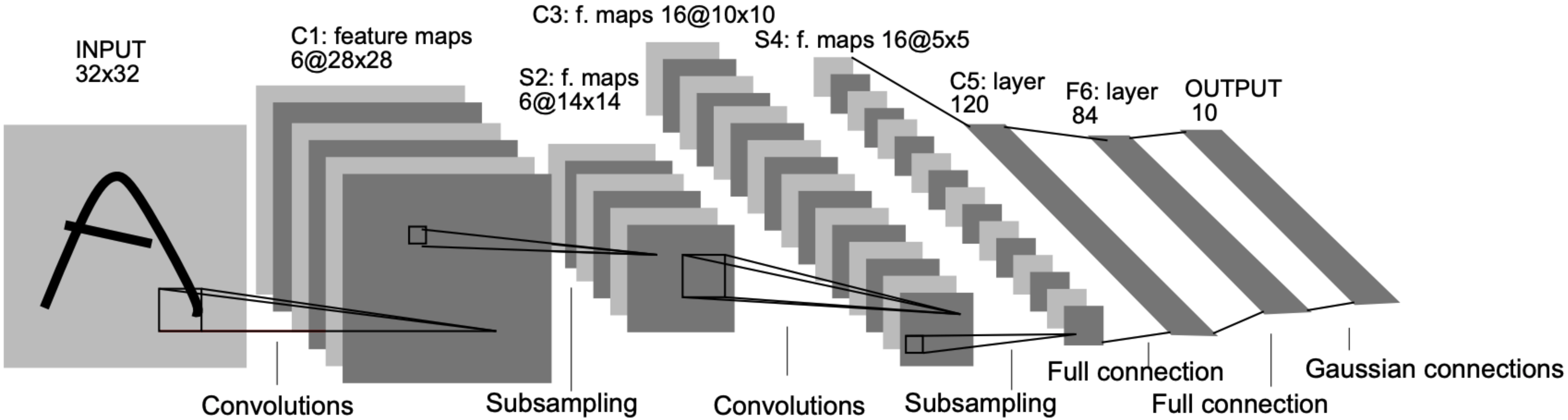


Fig. 1. Architecture of LeNet-5, a Convolutional Neural Network, here for digits recognition. Each plane is a feature map, i.e. a set of units whose weights are constrained to be identical.

LeCun, Yann, Patrick Haffner, Léon Bottou, and Yoshua Bengio. "Object recognition with gradient-based learning." In *Shape, contour and grouping in computer vision*, pp. 319-345. Springer, Berlin, Heidelberg, 1999.

ImageNet classification challenge 2012

AlexNet

Task 1

Team name	**Filename**	**Error (5 guesses)**	**Description**
SuperVision	test-preds-141-146.2009-131-137-145-146.2011-145f.	0.15315	Using extra training data from ImageNet Fall 2011 release
SuperVision	test-preds-131-137-145-135-145f.txt	0.16422	Using only supplied training data
ISI	pred_FVs_wLACs_weighted.txt	0.26172	Weighted sum of scores from each classifier with SIFT+FV, LBP+FV, GIST+FV, and CSIFT+FV, respectively.
ISI	pred_FVs_weighted.txt	0.26602	Weighted sum of scores from classifiers using each FV.
ISI	pred_FVs_summed.txt	0.26646	Naive sum of scores from classifiers using each FV.

ImageNet classification challenge 2012

AlexNet

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AlexNet

5 convolutional layers

2 fully connected layers

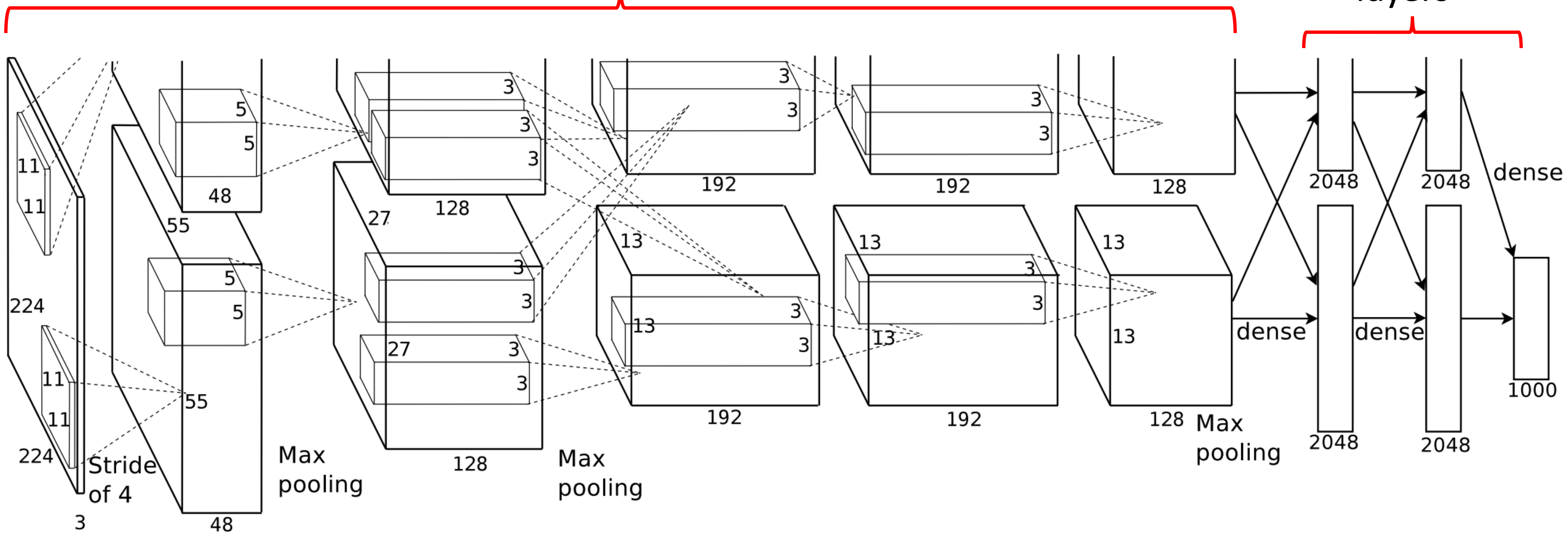
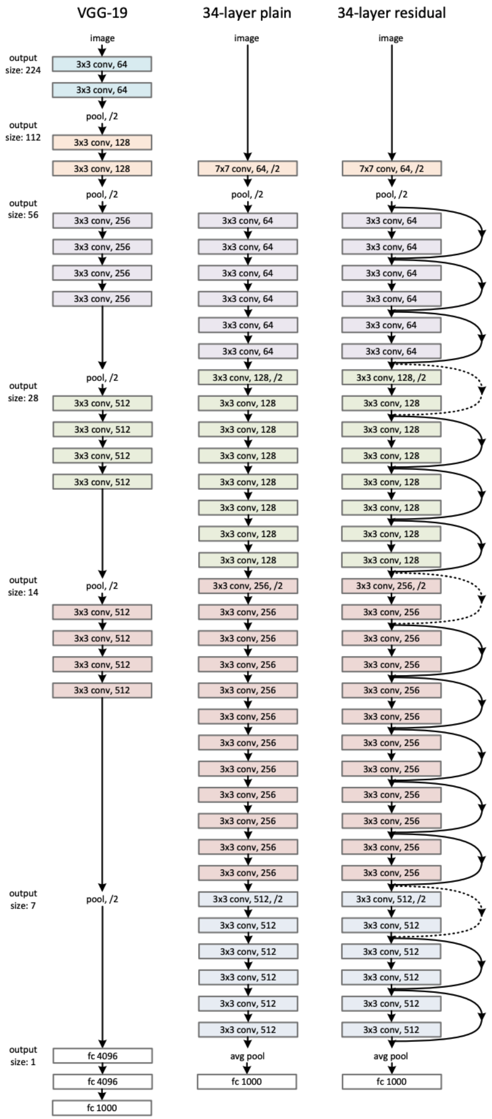


Figure 2: An illustration of the architecture of our CNN, explicitly showing the delineation of responsibilities between the two GPUs. One GPU runs the layer-parts at the top of the figure while the other runs the layer-parts at the bottom. The GPUs communicate only at certain layers. The network's input is 150,528-dimensional, and the number of neurons in the network's remaining layers is given by 253,440–186,624–64,896–64,896–43,264–4096–4096–1000.

Krizhevsky, Alex, Ilya Sutskever, and Geoffrey E. Hinton. "Imagenet classification with deep convolutional neural networks." *Advances in neural information processing systems* 25 (2012): 1097-1105.

VGG-19 and ResNet-34



[ResNet]

Feedforward networks

- Or multilayer perceptrons (MLPs)

$$f(w; x_i) = x_i'w + b \to y_i \quad \text{Linear model}$$

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$$f(w; x_i) = \underbrace{x_i' w}_{\text{Weight}} + \underbrace{b}_{\text{Bias}} \rightarrow y_i$$ Linear model

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Weight Bias

$$f(w; x_i)$$

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$$f(w; x_i)$$

If we composite another function

$$f_2(f_1(w; x_i))$$

Feedforward networks

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$$f(w; x_i) = x_i{}'w + b \rightarrow y_i$$ Linear model

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If we composite other functions

$$f_m\left(\dots\left(f_2\left(f_1(w; x_i)\right)\right)\right) \rightarrow y_i$$

Feedforward networks

- Or multilayer perceptrons (MLPs)

$f(w; x_i) = x_i{}'w + b \rightarrow y_i$ Linear model

Weight

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$f(w; x_i)$

If we composite
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1st layer

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final layer or output layer

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Linear model

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Feedforward networks

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Features must be extracted/generated (hand-crafted)

$$f(w;x_i) = x_i{}'w + b \rightarrow y_i$$

Linear model

Weight

Bias

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Feedforward networks

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$$f_m\left(\underbrace{\ldots\left(f_2\left(f_1(w; x_i)\right)\right)}_{\text{Hidden layers}}\right) \rightarrow y_i$$

Feature extraction/generation is not necessary
Features can be learned by hidden layers

final layer or output layer

Feedforward networks

- Deep:
  - Many compositional layers

Feedforward networks

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- Nonlinearity
  - Some functions f_i can be nonlinear

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e.g., activation layers

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Composition may not preserve convexity

Feedforward networks

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- Feedforward **Q**: why this name?

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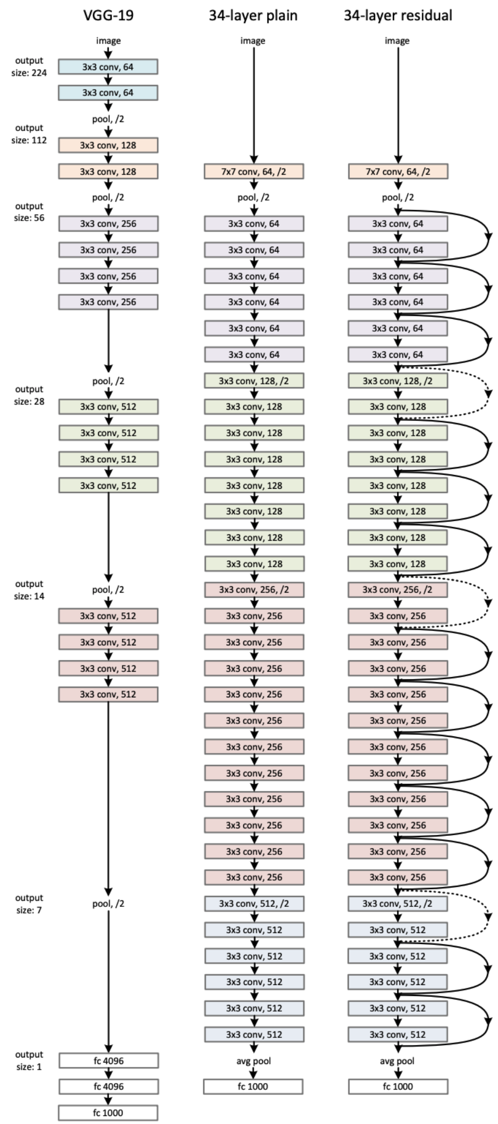

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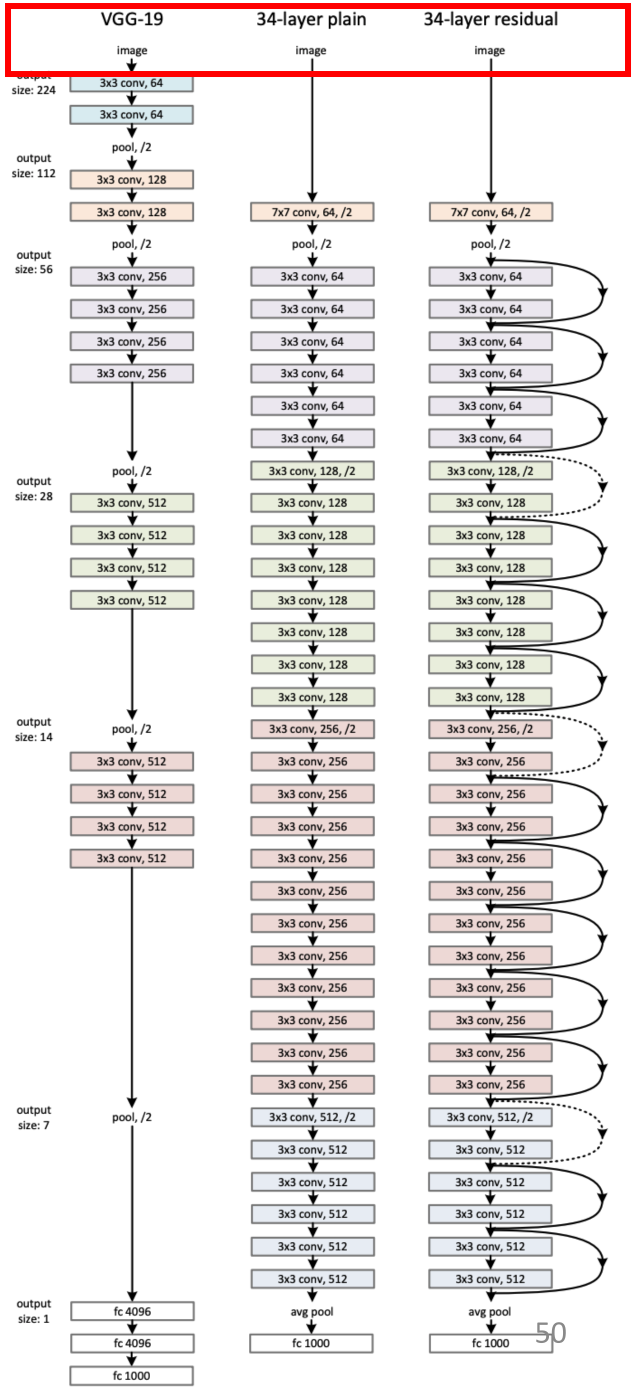

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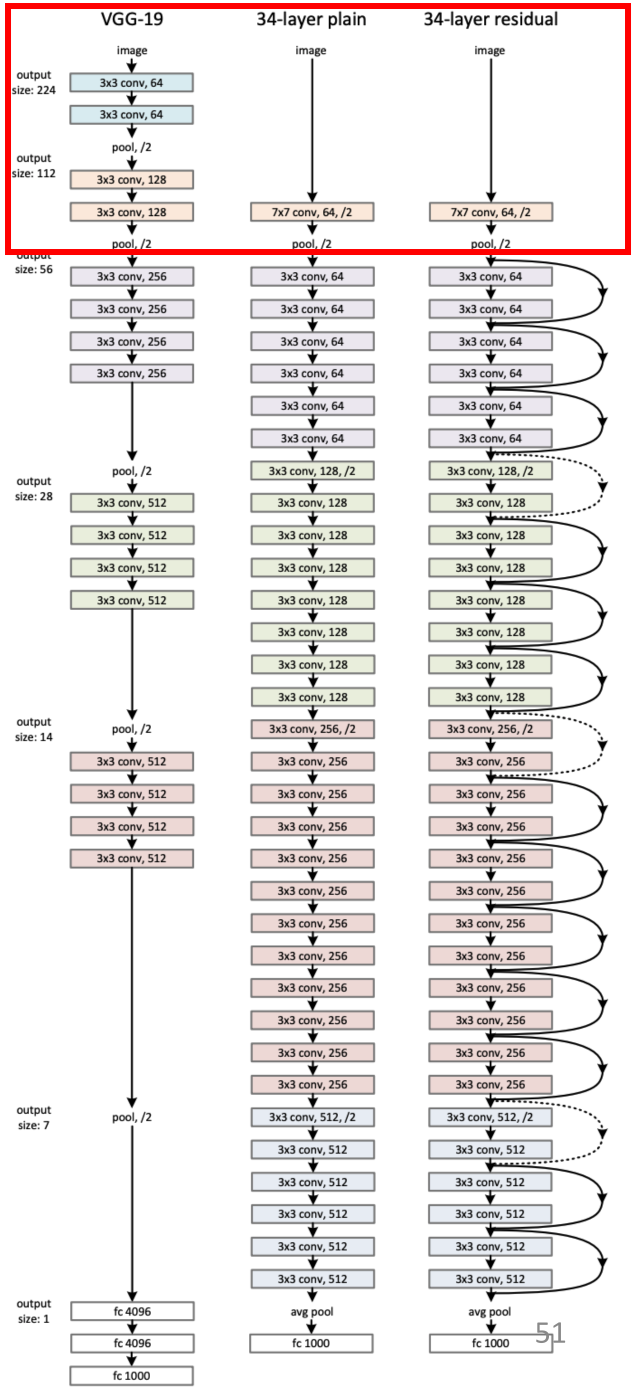

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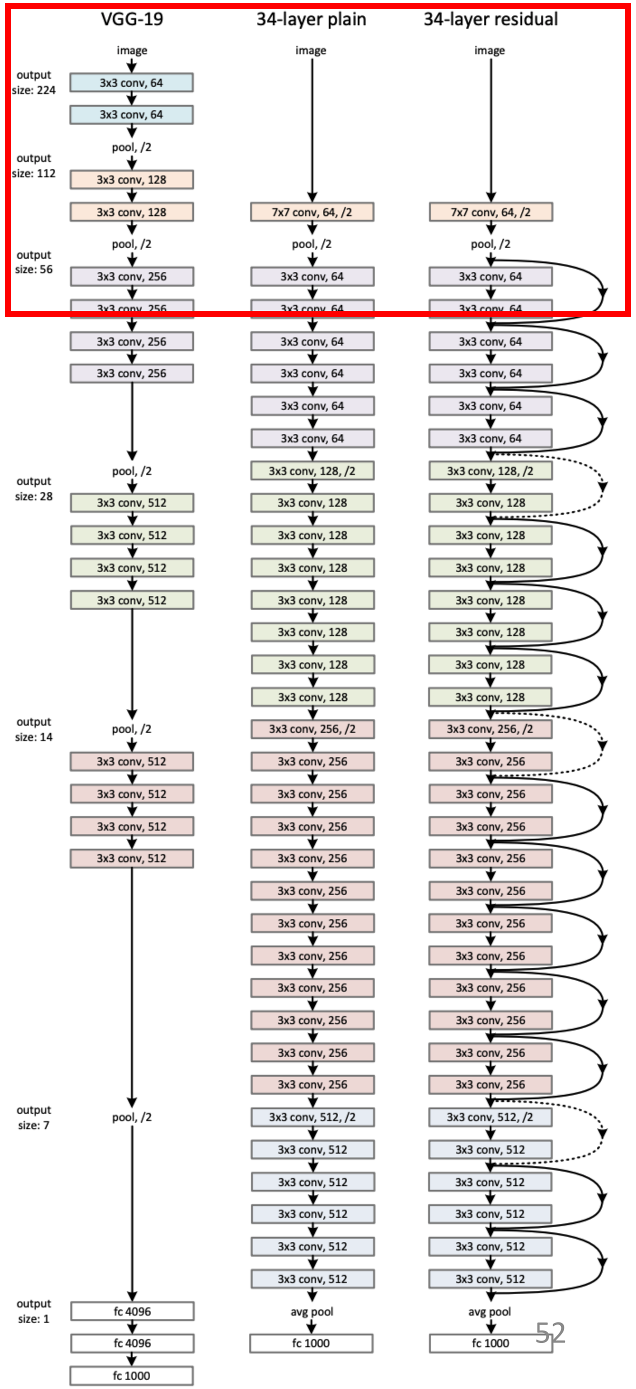
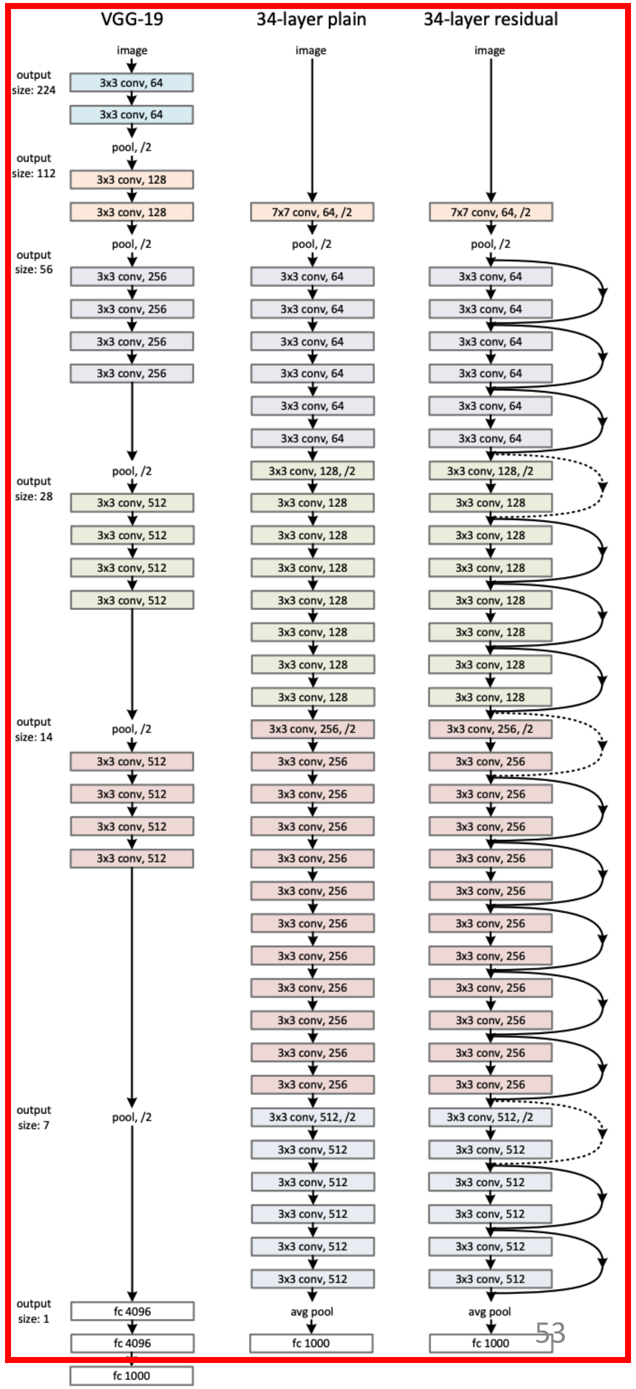
$$f_m\left(\ldots\left(f_2\left(f_1(w; x_i)\right)\right)\right) = \hat{y}_i$$



Feedforward networks

- Deep:
  - Many compositional layers
- Nonlinearity
  - Some functions f_i can be nonlinear
- Nonconvexity
  - Composition of functions
  - Some functions f_i can be nonconvex
- Feedforward
  - Information feedforward from input to output layer

$$f_m\left(\dots\left(f_2\left(f_1(w; x_i)\right)\right)\right) = \hat{y}_i$$



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Q: Any non-feedforward networks?

Feedforward networks

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- Nonconvexity
  - Composition of functions
  - Some functions f_i can be nonconvex
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  - Information feedforward from input to output layer

Q: Any non-feedforward networks?
Contains circles → recurrent networks

References

- [ResNet] He, Kaiming, Xiangyu Zhang, Shaoqing Ren, and Jian Sun. "Deep residual learning for image recognition." In *Proceedings of the IEEE conference on computer vision and pattern recognition*, pp. 770-778. 2016.